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Reported is a 1964 conference devoted to the problem of the low achiever in mathematics. Part I of the document presents five position papers concerned with motivation, Piaget's views on children's intellectual development, inplications of psychological research, the responsibility of school administrators, and business and industry cooperation with the schools. Part II offers six contributions describing current promising practices--programs in Baltimore and Fort Worth, mathematics materials specially prepared for noncollege bound students of average and below average ability, and an elementary mathematics laboratory. A series of recommendations are also included. (NH)

This report does not necessarily express the views of every participant at the conference and is not intended to be a statement of policy of the U.S. Office of Education. It is published by the Office of Education in the hope that it will stimulate widespread and responsible discussion of the issues that occupied the attention of the conference.

U.S. DEPARTMENT OF HEALTH, EDUCATION & WELFARE
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THE LOW ACHIEVER IN MATHEMATICS

(Report of a conference held in Washington, D.C., March 25–27, 1964, sponsored jointly by the U.S. Office of Education and the National Council of Teachers of Mathematics)

Prepared by
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FOREWORD

Early in the planning of the conference on which this report is based, it was decided to limit the participants to a small group of experienced persons with diverse backgrounds. A broad base of representation was deliberately planned, and it resulted in a variety of fresh ideas. The procedure was to consider first the sociological and psychological factors in low achievement, and then to study existing practices that appear promising.

A brief report of the conference was published by the National Council of Teachers of Mathematics (NCTM)[1] and was distributed to NCTM members. The preliminary report contained a summary of the recommendations and requested additional ideas for inclusion in this report. The response indicated extremely high interest in the topic.

The U.S. Office of Education deeply appreciates the help of the many individuals who contributed to this current report.

RALPH C.M. FLYNT
Associate Commissioner
Bureau of Educational
Research and Development

J. RICHARD SUCHMAN
Director
Curriculum and Demonstration Branch

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[1] *Preliminary Report of the Conference on the Low Achiever in Mathematics.* April 1964. 23 p.

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INTRODUCTION

WHY WE ARE CONCERNED ABOUT LOW ACHIEVERS IN MATHEMATICS

by

Harry L. Phillips

Specialist in Mathematics
U.S. Office of Education

The National Council of Teachers of Mathematics and the U.S. Office of Education are cooperating in this conference on the premise that the low achiever in mathematics is indeed worthy of a careful and considerate professional attention that has been unduly delayed in the preceding decade. Like it or not, we have suddenly awakened in a world which revolves around science, and it in turn rests on mathematics.

Our intent here is to consider the mathematical needs and proper instruction in mathematics for that category of youth referred to by Dr. Conant as "social dynamite"—those who possess no skill, who are unemployable and unschooled. These youths' estrangement from society has also been described by former HEW Secretary Ribicoff as "a terrible waste," and by Justice Goldberg as "potentially the most dangerous social condition in America today."

Our range of interest will include mathematics for those students who are potential dropouts, as well as for those who remain in school, but who, for one reason or another, exhibit a pattern of low achievement in mathematics. The major portion of research and conjecture on this problem suggests a re-evaluation of the education and training needs for the skilled and semiskilled citizen of the future. In such a re-evaluation, the U.S. Office of Education and the National Council of Teachers of Mathematics are firmly convinced that mathematics will be one of the more important disciplines.

The greatest single cause of unemployment is lack of education. As we try to cope with the problems of tomorrow, such as the expanding population, the rapid depletion of natural resources, the need for mathematically literate manpower, and the demands placed on the average citizen, we find the understanding of basic principles of arithmetic, algebra, and geometry more essential than ever before.

Throughout this conference, we want the participants to keep in mind that we are considering the lowest 30 percent based upon achievement in school mathematics, and not only the slow learner in mathematics. We are also very much interested in, and shall consider, those students who are capable but who are not achieving up to their capacity in mathematics. Specifically then, the U.S. Office of Education and the National Council of Teachers of Mathematics are interested in better instructional programs in mathematics for the low achiever for the following reasons:

1. Lack of both achievement and interest in the instructional program have generally been shown to be the principal reasons for students' dislike of school. Ultimately, if not corrected, these shortcomings result in school dropouts. We believe that because of the very nature of mathematics and because of some of the promising new methods of teaching this subject, there are hopeful prospects of overcoming both lack of achievement and lack of interest for many students in the lowest one-third of the ranks.

2. Training in mathematics, along with some degree of competence, gives a student a much broader choice of types of vocational training in the late secondary school or postsecondary technical training institutions. Two-thirds of the skilled and semiskilled job opportunities on the labor market today are not available to those who lack an understanding of the basic principles of arithmetic, elementary algebra, and geometry. Basic mathematical understandings are also essential to adult retraining programs for the unemployed.

3. The mathematical community and mathematics educators are very proud, and rightfully so, of the advancements made over the past 15 years in the mathematics curriculums and in teaching at all levels of education. At all levels, however, the emphasis and attention have been directed toward the above-average mathematics achiever. This has been true of the study groups which have spearheaded curriculum revisions as well as of the commerical publishers. We believe it is now time to show some consideration for the low achievers. Even those in the "mid-ranks" in mathematics achievement have been somewhat overlooked in the push for maximum development of the college-able student. We also believe that if a program designed to raise the mathe-

matical achievement level of the lowest 30 percent were to succeed, it would set off a chain reaction that would affect students in the middle range of achievement as well.

4. The student of low general ability, who is also likely to be a low achiever in mathematics, may, with the proper program and improved methods of teaching, be able to enter the labor market less vulnerable to lurking unemployment possibilities. At the very least, a better understanding of elementary mathematics will make retraining much easier and more acceptable in later life.

5. Success or measurable achievement in mathematics has a close correlation with increased achievement in other disciplines. We believe that other disciplines may profit from the patterns evolved as mathematics programs for the low achiever become more effective.

PART I

POSITION PAPERS

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HOPE, DELUSION, AND ORGANIZATION: SOME PROBLEMS IN THE MOTIVATION OF LOW ACHIEVERS

by

Jules Henry[*]

Since discussions of human motivation usually deal only in summary fashion with physiological motives, including survival, it is not possible to use current motivational theory when the context is one in which physiological motives, particularly survival itself, are paramount. This, however, is precisely the environment of many low achievers. Motivational theory includes the concept of goal, and uses such propulsive words as "energizes," "activates," and "moves." In the environments from which low achievers largely, though by no means exclusively, come, such ideas are hard to find. In reading about human motivation one is struck by the fact that what the motivation researchers have in mind, the hidden parameters of their thinking, are largely those attractive to the researchers. For example, in a recent book the following are stated to be "secondary, learned, social, or psychogenic motives. . . . To strive for social acceptance or status, to work to write a symphony or climb a mountain, to try to keep the schools segregated or to integrate them, to want to complete college or understand human behavior . . . saving for a trip abroad, working to get ahead, buying a new car or reading a book . . . heroism, martyrdom, artistic production or religious asceticism.'"[1] While, strictly speaking, these are a mixture of ends and means rather than motives, the selection of examples illustrates the problem viz, that the parameters of research in motivation are largely middle-class and elite; and from this point of view motivation is a unique psychological quirk characteristic of the middle and elite classes only. In such a context then, most lower-class children could not be said to have any motivation.

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[*] Dr. Henry is professor of anthropology and sociology at Washington University, St. Louis, Mo. His studies of schools and schoolchildren started in 1953. Recently he became involved in the study of low achievers because of his connection with the study of a public housing project in St. Louis occupied mostly by extremely poor Negroes. At the moment he is directing a study of Negro kindergartens in that area and of the families of the kindergarten children.

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Under a National Institute of Mental Health special grant[2] a research team from Washington University has been studying a housing development in St. Louis inhabited by very poor Negroes. One of the points brought out in our discussions of the tenants in the project is the tendency to apparently random behavior. Since we arrived at the notion of randomness through the impression that in their general conduct of life many of the households in the project did not conform to middle-class ways, we understand that the impression of randomness is relative—relative to middle-class behavior. Bearing this in mind, we perceive that with regard to space, time, objects, persons, and so on, the behavior of the people who live in the project, failing to follow the patterns of organization characteristic of middle-class society, gives the impression of being random, i.e., lacking pattern and therefore lacking predictability from our (i.e., middle-class) point of view. The question arises, of course, whether the behavior is random from their (i.e., dwellers in the project) point of view. We have the impression, however, after 7 months of field work with about 50 families, that they look upon one another's behavior in somewhat the same way. The distinction they make, for example between C.P. time (colored people's time)and W.P. time (white people's time) is suggestive; for according to C.P. time, one never knows whether an event will occur when scheduled, while according to W.P. time, events scheduled for a given hour always occur at that hour.

The opposite of randomness is *organization*. The further down we go in the vertebrate phylum, the more the organization of behavior is determined by genetic mechanisms, although it seems that at no point is behavior in the vertebrates determined exclusively by them. In *Homo sapiens*, of course, it is difficult to show that any intraspecific (i.e., social) interaction is determined by innate mechanisms only; and the word *culture* has been chosen to designate what is determining in all the behavior of *Homo sapiens*.

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[1] Bernard Berelson and Gary Steiner, *Human Behavior*. New York: Harcourt Brace and World, 1964. pp. 240, 241.

[2] Grant No. MH-09189: "Social and Community Problems in Public Housing Areas."

Yet *culture* has a certain phenomonology (i.e., a system of interrelated, relatively standardized component perceptions) that is different for every culture; and this phenomonology exercises the same control over behavior in *Homo sapiens* that genetic mechanisms do in the lower vertebrates. We have a fairly accurate impression of what these component perceptions are in the middle and upper classes in our society. The notions of achievement and security are familiar. We know also that these notions *organize* the behavior of the middle class. The point of view in which achievement and security are salient is complemented by the opposites: failure and insecurity. This logic enables us to say that since achievement and security organize (i.e., make nonrandom) the behavior of the middle class, their absence will result in random-appearing behavior. We may thus say that wherever, in our culture, achievement and security cease to be components of perception, behavior will appear random. This is the condition in many households in the project.

We have barely begun, however, analysis of the phenomonology of the culture of the middle class. We have yet to deal, for example, with the problems of *hope, time,* and the *self*. An important modality of achievement is *hope* and a central modality of hope is *time*. Achievement has a temporal dimension, for it means "some time in the future." Even when we say "Billy no longer wets the bed" we mean that *in the course of time* Billy stopped wetting the bed, although he used to do it; and nearly every parent hopes that his child will stop wetting the bed. We can therefore imagine that the parent who has no hope has no conception of his child's stopping wetting the bed. This behavior in his child will therefore have dropped out of his conception of the *organization* of his child's behavior. But in a broader view, the person who has no hope of achievement or security will have *no conception whatever of the organization of behavior* (relative to middle-class behavior) at all. We can say, perhaps, that the households of the project have no hope relative to middle-class orientations and that their behavior therefore appears random (i.e., unorganized) to a middle-class observer.

What then is the phenomonology of no hope? Though many of the households of the project have no hope, they nevertheless wish to stay alive. Under these circumstances the *concrete factors that keep them alive* (i.e., that save them from death) *move into prominence,* and other perceptions (i.e., many perceptions oriented toward middle-class organization) are not present in their awareness. What becomes salient are the factors that make a direct contribution to *survival:* perceptions of objects, persons, and conditions that make a direct contribution to the *possibility of death,* of the *violation of the person,* and of the violation of *material factors contributing to survival.* In this context, the entire orientation toward objects must change, for if objects do not contribute directly to survival (to protection against death) they tend to be unimportant. For many dwellers in it, the project is a culture oriented largely toward survival (toward *flight from death.*). In this context disregard of many modalities of objects, for example, their arrangement in the house, becomes institutionalized, i.e., a way of life.

Let us consider now important conceptions borrowed from Heidegger.[3] When the middle-class person thinks about himself (or his self), he says, "I *used* to be that way; here is the way I am *now,* and I *hope* I *will change* for the better." These perceptions of the self have a past, a present, and a future; and from this middle-class view of self (*Dasein*) Heidegger derives our conception of time. Let us consider now the condition of people for whom this comparison does not exist, i.e., people who do not view the self as being in a state of change. Obviously, if the self is perceived as being in a state of change from what *used to be* to a state of what *hopefully will be,* a certain organization of activity must ensue; for the passage consciously and determinedly from what used to be or what is now to a state of what will be requires an organization of existence that will bring these changes in the self to pass. But if this temporal modality of the self drops out, the organization in the activities of the middle-class self will not appear. Hence, having no perception of temporal dimensions of the self, such a person exhibits apparently unorganized, or random, behavior. What remains is the *survival self,* the self that is in flight from death. This self then becomes preoccupied with activities that give it *the most intense sensation of being alive;* it is a self that must, at every moment, literally feel its life. This is the condition the middle-class sociologists contemptuously call "hedonism." But it is not hedonism; it is merely flight from death.

It may be helpful now to look at the apparently random behavior of the dwellers in the project in the metaphorical context of entropy. In thermodynamics, entropy is a measure of randomness and hence of loss of organization. We can say, therefore, metaphorically speaking, that entropy is maximized in many households in the project. In thermodynamic theory, however, entropy increases in closed systems only, that is, in systems that cannot receive energy from outside the system. The isolated quartz crystal is a common example. Before attempting to view the project (metaphorically) as a closed system, let us look at its opposite, a middle-class dwelling area. There we perceive that the members have relatively free access to the major sources of cultural (including economic) stimulation and that households are therefore able to maintain organization (entropy is at a very low level). In the project, however, the households do not have available such sources. And here is the paradox, for what prevents the dwellers in the project from having access to the major sources of cultural stimulation is their randomness and absence of hope, and what created their randomness and lack of hope in the first place was inaccessibility of the cultural resources. We can therefore say that since the households of the project are alienated from middle-class culture, their entropy can only increase.

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[3] Martin Heidegger, *Being and Time.* Heidegger, of course, has no interest in culture and even less in social class. The discussion merely borrows some of his fundamental conceptions of *Dasein* as a temporal being.

It goes without saying, of course, that the self of many among the middle class is also in flight from death, but since in the middle class the orientation toward achievement is the lens through which existence is perceived, and since they have been taught the possibility of hope, they *fly from death toward achievement,* sustained by hope. This way of seeing life and this way of being sustained are not available to many of the dwellers in the projects—to many dwellers in the slum.

We thus come to the realization that hope is a *bounding* phenomenon; in the sense that hope separates the free from the slaves, the middle and upper classes from much of the lower class, the hopeful from the hopeless. We thus come to an even more unexpected conclusion, to wit, that time, space, and objects exist in an environment of hope.

The Delusional System

Since the project is isolated from the mainstream of social and economic life of the city as well as from the white community, the occupational classes of the Census Bureau do not apply to the tenants. Since most of the people work at interstitial jobs or work as domestics, and since their employment is precarious and poorly paid, resources are scarce. Though the project is almost literally a "City of Women," for a very large number of husbands are transients or have deserted, the women talk about husbands as if they existed, and the unemployed men talk as if they had jobs. In addition, there is the constant effort to build one's self up, by inflating one's self, by spending one's money on very expensive clothes and by getting the better of another person. Thus, achievement is delusional also. In view of this, one can understand the remark of a white school teacher working with poor Negro children, that they are not interested in solid accomplishment but rather in showing off. All of this must interfere with learning in school or even taking school seriously. Thus we see that just as violent rejection by a parent tends to create delusional fantasies in his children, the casting out of the Negroes by white society results in the development of a social life so saturated with delusion that *delusional* achievement becomes the *real* achievement.

In School

I have spoken so far of some general characteristics of the culture from which one type of low achiever comes. The child, however, is a low achiever in a particular setting, the school; and the classroom has its own social dynamic into which the child's lack of organization and his tendency to delusional achievement fit perfectly.

It is clear that children from disorganized backgrounds cannot create unity in a classroom; and under these circumstances, the teacher can work only with those who somehow have enough of the necessary motivations. We therefore sometimes see a harassed teacher working only with these children, while letting the rest of the class carry on in a disorganized and disorganizing way. Thus the *universe of participation* in the classroom work becomes very small. I now give some extracts from the observation of one such classroom. The teacher is a white woman and the children are Negro and white. It is the sixth grade and the time is 11 a.m.

The teacher was leaning over Paul's desk helping him with arithmetic. Both her hands were on the desk. Paul was in his seat, his head on one hand, focusing on his paper. Irv and Mike were watching. Across the room Alice was talking to Jane, and Joan was talking to Edith. Behind me I could hear a girl seated at the work table whispering and talking. Nearby there was pushing and shoving in a group of boys—Alan, Ed and Tom. Tom got out of his seat, made a wad of notebook paper and tossed it in the air several times. He intended to catch it but dropped it on the floor. As he bent to pick it up he dropped his pencil on the floor and kicked it four or five feet across the floor. He picked up the pencil and threw the wad at Alan. Alan turned to the observer, grinned, waved his hand, and said, "Hi." The teacher took no notice. . . . Tom and Ed suddenly slammed their desks shut, got up and walked out noisily. As they left, Lila and Alice hurried to catch up with them and walk out. Just before they left the room, Tom turned and called back to Lila, saying, "Come on Lila." A girl called out, "Good-bye," and Josephine waved good-bye.

With her back to the door the teacher was talking to Jane. She stood by Jane's desk with her hands on Jane's desk, while they both looked at the student's work.

Irv got up from his desk and walked over to stand near Joan's seat. Josephine got up and pushed Irv and he pushed her. The teacher turned from Jane's desk, walked over to Josephine and Irv and stepped between them. She flushed as she squeezed Irv's shoulder and pushed him away. Half pushing and half leading, she got Irv back to his seat and forced him down into it. Josephine, very cocky, stood beside her desk, hands on hips. The teacher looked at Irv for a minute and then walked back toward the work table. Irv sat in his seat but a minute, and the next thing I knew he was down on the floor. Mike and Paul were laughing at him, and Mike seemed to be kicking at him. Irv got up, walked behind Mike's desk, clamped a headlock on Mike and tried to pull him out of his seat. . . .

11:12 a.m. The teacher is helping Mike. She stands beside his desk, looks down at his work but does not touch him. Lois has walked across the room to talk to John. Her voice is loud, but the noise in the room is so great that it is difficult to hear her. Lois and John leave the room, passing in front of the observer and saying, "Excuse me." . . .[4]

The teacher let the other children do largely what they pleased, while she gave herself to the three who were able to resist the general *strain toward disorder* and do their work. This phenomenon of *partial withdrawal* may occur under any circumstance where a single individual attempts to cope with a disturbed environment; and I have seen it in institutions for emotionally disturbed adults and children. The *partial withdrawal syndrome* is not, however, a function of the children and teacher only, but also of the dynamics of the social situation. For example, in the selections just read, the children had a choice of seeking status with peers or with the teacher; for in this environment, status with the teacher is viewed by some as incompatible with status with peers. Hence most of the children abandon their lessons in favor of "messing around" with peers. *Self-destructive status choice* can be seen in varieties of disturbed ("split") environments—usually environments in which *the choice is between a peer group and an authority figure*, but especially where hope does not tip the balance in favor of the teacher and self-preservation. We see this sometimes in the case of Negro children in integrated schools, where, in their anxiety to be accepted by the white children, they will follow the whites in "messing around," instead of yielding to the authority of the teacher. It is likely then that the seriousness of the problem of *self-destructive status choice* is increased when the disturbed situation has special attractions for the children. Thus in a coeducational class the valence of the peer group is increased by sexual attraction; and in a biracial class, valence of peers may increase for the Negro children.

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[4] Reproduced by courtesy of The Youth Development Project of The Greater Kansas City Mental Health Foundation.

Summary and Conclusions

In attempting to examine the achievement problem in very, very poor Negro children, I suggest that they lack both hope of achievement and fear of not achieving and that they come from a culture lacking the characteristics of order fundamental to the achievement-oriented middle-class culture. Specifically their homes are physically and personally disorganized, life does not run on a time schedule, and so on. Thus, emotionally and cognitively, they lack the structure on which a conventional educational system can build. When 30 to 50 such children are placed in a classroom run by 1 teacher, the result is bound to be disorganization, from which the teacher will select those elements of order suited to her task: she will teach the children who are teachable and let everybody else go. Meanwhile, even the children who want to learn are under tremendous pressure from their peers to give up. Thus the motivation of the low achiever is not a demon locked up inside the child, but is at every moment, especially in school, subject to manipulation by the peer group as well as by the teacher. One might urge, therefore, that, in considering improving the motivation of the child, one should also improve the school as a social system.

Proposals

A program of total reconstitution is needed

Filling the cognitive gap.—I have pointed out that because of the disorganization of the environment, the basic perceptual frames of these children do not seem to have been properly constituted: houses and people are in a constant state of disorder and things do not run on time. Yet these children are supposed to master mathematics, the central idea of which is order. I would urge that in preschool these children be formally introduced to fundamental shapes and categories: insidedness and outsidedness, roundness, straightness, flexibility, rigidity, transparence, opaqueness, motion in a straight line, in a circle, rocking motion, motion that rolls but moves in a straight line (like an automobile, for example). Problems in the articulation of gears and movement in several planes at once (as, for example, in a manually operated eggbeater). Things that flow (water and sand—as in and out of a sand pail), things that "shove along" (like blocks), and so on. Through planning basic experimental frames, much can be done to build up the necessary perceptual competence in these children prior to their entrance into elementary school.

Emotional calming down.—These children often come to school unfed after wretched nights torn by screaming, fighting, bedwetting, etc.; often they have not slept because of cold and rats. For such children to start at once the routine work of the average elementary school class is impossible, for not only are they hungry and sleepy, they are emotionally upset. It is therefore proposed that teachers be trained[5] to deal with the problems of these children and that such trained teachers have breakfast with the children in school. It is expected that the school will furnish the food. Of course, one does not have to wait until the teachers are trained in order to bring children and teachers together at breakfast. The purpose of the breakfast is twofold: (1) to feed hungry children and (2) to bring teacher and pupil together in an informal atmosphere before the pupils are placed under the strain of classroom constriction.[6] *It is essential that the teacher be present,* so that the students meet her under the calming conditions of friendly eating together.

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[5] As in the Youth Development Project of the Greater Kansas City Mental Health Foundation.

[6] One such program in Kansas City worked an immediate sharp improvement in attendance and all other areas of behavior as well as in school work.

Under these circumstances the personalities (the "egos") of the children, badly battered by their night's ordeal, will be "reconstituted." The more the teachers know about the emotional management of these children, of course, the better.

Expansion of the universe of participation.—This may be done either by reducing the size of the classes or by increasing the number of teachers in the classrooms to two or three. These have to be trained personnel, who know the subject matter of the lessons. The supernumerary personnel could be teachers-in-training, members of the domestic Peace Corps—whoever is in a position to learn the material.

STAGES IN THE CHILD'S INTELLECTUAL DEVELOPMENT: PIAGET'S VIEWS

by

Lydia Muller-Willis*

At this time, when school curriculums are being revised, thoroughly reevaluated, and even rebuilt, most of us turn to the Swiss psychologist Jean Piaget, who has spent his life doing research with children and has truly become an international authority in the field. The most comprehensive account of Piaget's work, up to 1960, has been done by John H. Flavell.[1]

Piaget has studied his own children and has noticed that, up to 18 months, that is, before the appearance of language, they have to learn the permanence of the object. The very young child thinks that if an object is hidden, it does not exist any more. As the child gets somewhat older he tries to find it, thus showing that he has acquired a sense of the permanence of the object. Piaget has called the first period of the child's development the *sensori-motor* stage. Piaget sees *perception* as an example of continuous development in contrast to *intelligence*, which follows a definite sequence of stages. Each intellectual stage implies a period of formation and attainment as well as the starting point of a new evolutionary process. The order of succession or sequence is constant, but the chronological ages at which children reach these stages may vary.

Until the age of six or seven, there is a second stage, which is called *preoperative*. At this stage, the child may have the notion of conservation of an object, but he does not yet believe in the conservation of a collection of objects. For example, if you give two rows of blocks to the child in one-to-one correspondence:

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*Dr. Muller-Willis is a graduate of the University of Geneva, where she studied under Jean Piaget. She is author of the book "Research on the Understanding of Algebraic Numbers by Children," and is currently a research associate at the University of Minnesota, working on the Minnesota School Mathematics and Science Teaching Project with Dr. Paul C. Rosenbloom, author of the paper that follows this one.

[1] John H. Flavell, *The Developmental Psychology of Jean Piaget.* New York: D. Van Nostrand Co. 1963.

x x x x x x x
x x x x x x x

and then space the blocks of the second row differently:

x   x   x   x   x   x   x

the child says there are more blocks in the second row.

Thought at this stage is largely based on perception, and usually one aspect (or one dimension or one relation) is considered at the expense of the others. If the same amount of liquid is poured into two similar glasses and then the liquid from one of them to a taller glass, the child says there is more liquid in the taller glass than in the other. Here he considers just the aspect of height. Also at this stage the child has difficulty in conceiving of or understanding the point of view of others, and this tendency has been called egocentrism.

During a third stage, lasting from 6 or 7 years to 11 or 12, the child considers two or three dimensions simultaneously. This stage is called the *concrete* stage, or the stage of concrete operations. These concrete operations deal directly with objects, and, the child learns to make classifications and seriations.

It is interesting to note that during this stage the concept of substance is first acquired, then the concept of weight, and finally the concept of volume. To study the development of these concepts Piaget uses (among other materials) two balls of plasticine or clay. He changes the shape of one of them, making it into a disc and a sausage, etc., and asks the child if the disc, say, has the same amount of clay as the ball or if it weighs the same as the ball. For the concept of volume he drops the clay balls into water, takes one out, changes its shape—into a sausage, for example. He then asks the child if the level of the water in which the sausage will be dropped will be the same as the level of the water in which the ball is resting.

Finally, at the age of 11 to 12, the preadolescent becomes capable of reasoning not only on objects or actions themselves, but also on operations expressed by propositions. This is the stage of *formal* operation, that is, of abstract thought or of reasoning by means of pure symbols without perceptive data. This is the area of hypothetical-deductive thought which goes beyond the immediate, perceptively given reality. In contrast to the child, whose thought is still dependent upon manipulation of concrete objects, the adolescent is capable of forming hypotheses and of deducing all the possible consequences from them. These new abilities open up unlimited possibilities for the adolescent to participate constructively in the development of science, as long as the environment offers him opportunities to practice as well as a favorable intellectual atmosphere.

The development of intelligence in the child is very important to teachers. Indeed, the child will show *much interest in discovering* a law (or the answer to a problem the teacher has laid out for him) *which corresponds to a structure he has already mastered* (such as a law of seriation), while the child of a lower level of development will not show this interest. According to Piaget, the child's motivation will be twofold: first, affective (that is, he will bring emotional strength which will facilitate learning) and second, cognitive (that is, he is already in possession of the preceding learning structures necessary for the new learning, and may even be in the process of building the new law).

Before we start on curriculum and technique, however, I would like to give you Piaget's explanations for the transformations which take place from early childhood to adolescence. He gives four factors:

1. *Maturation of the nervous system.*—This first factor does not explain everything, but it should be remembered that the order of succession of the stages is constant, though the chronological ages will vary.

2. *Experience with objects of the physical world.*—Again, this factor does not explain everything for several reasons:

   (a) Some notions, such as substance, are acquired at the beginning of the stage of concrete operations, and others, such as weight and volume, not until later on.

   (b) There are two kinds of experience: (1) the physical experience, which is an action or an abstraction from the object; this kind of experience leads to such statements as "This watch is heavier than this pencil," and (2) the logical-mathematical experience, which is knowledge gathered from actions on the object. A good example is that of the child's counting pebbles, lining them up in one direction, then in the reverse direction, and finding that the sum is independent of the order. In this case the child himself introduces order.

3. *Educational transmission, such as language.*—The child can receive valuable information only if he is ready to understand it.

4. *Equilibration, or balance among the first three factors.*—There is a succession of levels of equilibration such that the attain-

ment of level two is possible only when equilibration has been obtained on level one. For example, when the ball of clay is changed into a sausage, there are probably four successive levels: first, the child thinks just about one dimension such as length, without taking width into account. If the examiner continues to lengthen the sausage, the child at a given time says "It is getting too thin," and he thinks about the width. Then, on a third level, he wavers between the factors of length and width, he discovers their solidarity and says "When you make it longer it becomes thinner . . . consequently, it is the same thing," and this is his equilibration process coming into play.

Some psychologists have tried *bypassing the stages* but their results show that little can be done in this respect. In the case of *physical experiments* on such phenomena as conservation of weight, using a scale as external reinforcement, some learning takes place before the expected age. But in the case of *logical-mathematical* experiments on such phenomena as constancy despite apparent change, the child resists learning. Piaget thinks that, as far as physical experience is concerned, learning is indeed possible at an earlier date, but not in the case of logical structures, such as transitivity of weight, which is only obtained by autoregulation and not by external reinforcement. However, learning of this type of structure is possible if the structure is built on simpler or more elementary structures. As we have seen, many of the concepts that adults take for granted have to be slowly constructed by the child.

If better teaching is to help the low achiever in mathematics, then *better teachers* are needed. By "better teachers" we mean teachers better prepared psychologically. It is essential for the teachers to know the operational level of the pupil. If teaching is *active* enough, direct observation will permit judging the child's level. If teaching is not active enough, the teacher must have some simple functional tests at his disposal so that he can judge his pupils' level. I should like to point out that Piaget's definition of the word "active" is twofold, pertaining both to the child's *manipulation of objects* and to *cooperation among pupils*. Teamwork helps children to learn from one another.

One of the difficulties is that teachers know much more about children than psychologists do, and they often think that the experiments psychologists are conducting are artificial or have no connection with teaching in a classroom. Teachers should be given a chance to repeat some of Piaget's experiments, or better yet, to participate in a research team and to do work themselves on some specific problems of low achievers in mathematics.

In the active method the role of the teacher consists essentially of providing the pupils with the necessary materials and of creating situations which lead the child to generalize. The heart of the active method lies indeed in the *spontaneous construction of the operations by the child.* The child is endowed with the tendency to spontaneous activity and the teacher can encourage its development. Thus the pupil solves problems by manipulation and through trials, at a pace suited for his intellectual ability. The child puts the emphasis where he has most difficulty and the crucial points for his understanding are put forward. Hans Aebli,[2] after experimenting in schools with the method I just sketched, concludes that the extra time spent on personal research, trial and error, and self-construction of knowledge is worth the effort at all levels of teaching. He thinks, however, that the use of concrete manipulations at the upper levels of primary school and in high school does not justify the slower rhythm of teaching except for the less gifted children.

The active method also has the advantage of giving the low achieving child a chance to learn to *verify*, to decide what is right and what is wrong, and this by doing a real verification himself! The role of the teacher consists of giving the child the instruments which will permit him to decide what is right and what is wrong, rather than correcting the child by verbal and prohibitive methods. The child learns better by seeing the data and noticing changes or constancy, etc. One should make him feel that he is capable of dealing in his own way with the problem at hand. One should not tell him too much, but let him figure it out himself. Every answer the child gives is significant, and it is important for the teacher to discover the process underlying the wrong answers as well as the right ones.

This material, or external, verification is the first reason for the child's reinforcement, or motivation. The second reason is the internal verification, that is, the pleasure of discovering coherence in areas which until then appeared poorly related. For example, the child is greatly satisfied when given a program in which he can pass from arithmetic to geometry. The *materials* used with low achievers should be as interesting as possible. They can often be presented in the form of a game that stimulates the child.

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[2] Hans. Aebli, *Didactique Psychologique, Application à la didactique de la psychologie de Jean Piaget. Neuchâtel:* Delachaux et Niestlé. 1951.

Classrooms are usually made up of children of the same age. One could conceive classrooms in which a *mixture of ages* would make for better cooperation. This would be useful mainly for older children, especially those suffering from affective disorders causing them to think that they do not understand mathematics. By explaining to a younger child, an older child of this kind might gain confidence in himself. (Among adults the same is true; professors often learn more than listeners!)

In any classroom there are differences in the level of development of the children. Low achievers often need more concrete manipulations than others. Some children are slower in discovering the principle involved in the materials presented to them. A practical technique seems to be the *partial individualization* of teaching. The teacher can allow the pupils who are further ahead to deal with individual or collective work corresponding to their intellectual level, while he watches and helps the low achievers to manipulate actively and concretely and to discover the notions they need.

IMPLICATIONS OF PSYCHOLOGICAL RESEARCH

by

Paul C. Rosenbloom*

I have been asked to comment on the work of Dr. Muller-Willis on the implications of Piaget's research for curriculum development. Piaget's identification of the stages in the development of intelligence provides a useful framework for thinking about teaching mathematics to low-ability students. These stages are:

I. Sensori-motor intelligence (up to age 4).
II. Intuitive thought (ages 4–7).
III. Concrete operations (ages 7–11).
IV. Formal operations (ages 11–15).

The indicated age ranges are approximate averages for children in our culture; for the development of particular concepts, the average ages at which children attain the various stages, may differ considerably.

According to Piaget's theory, the mechanism whereby a child passes from one stage to the next is a dynamic process of approach to equilibrium between two tendencies in the child's thought: assimilation and accommodation. "Assimilation" is the incorporation of objects into the child's patterns of behavior, the changing of the signals the child receives from his environment to fit the mental structures he already has. "Accommodation" is the modification of the child's patterns of behavior to fit his environment, the changing of his mental structures to fit the signals he receives from his environment. If a child calls a cloud a bear, he is "assimilating" his perceptions to the mental structures he already has. When he learns to classify clouds as nimbus or cumulus, he is "accommodating" his mental operations to his perceptions. We may think of the children we are concerned with here as those whose development through these stages has lagged significantly behind that of most other children of the same age. As examples of low achievers, we are talking about 7-year-olds in stage I or 10-year-olds in stage II or 13-year-olds in stage III.

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Before going into more detail, I should like to make a few general points:

1. *To understand the behavioral criteria for identifying the stage of a child requires a much better mathematical and psychological background than most teachers possess.*

Piaget uses freely such mathematical concepts as group, topological space, and coordinate system, with which most teachers and psychologists are unfamiliar. He also uses a physcological terminology of his own invention. His few expositors have been psychologists who do not themselves understand clearly the mathematical concepts with which Piaget deals.

2. *Bad teaching or writing may make a bigger difference for slow learners than for average students.*

An average or bright child may be able to figure out a problem for himself in spite of a muddled explanation, but a dull child has very little hope of doing so. This is confirmed by data from SMSG experimentation by the Minnesota National Laboratory, which indicate that slow learners benefited more from SMSG than average students, as compared with the achievement of similar students in conventional curriculums. My hypothesis is that a clearer exposition, with greater attention to precision of language, made a bigger difference for the slow students.

3. *In most schools the teachers of slow learners are either the lowest in the pecking order in their respective departments or specialists on low-ability students with little special knowledge of mathematics.*

An effective curriculum for slow learners will have to be developed by the few highly imaginative people that can be recruited, and it should be written in a form which teachers of low mathematical competence can handle.

4. *Mathematics and reading are the key subjects for making low-ability children employable.*

This is indicated by the reports of the U.S. Department of Labor on the retraining of workers.

5. *In order to make these children employable, they must develop some abilities which cannot be duplicated by machines.*

Even animals, whose brains are not highly developed, such as pigeons, can learn tasks which are extremely difficult to program for computers. For example, they can learn to track a moving object. Many mentally retarded humans can recognize printed letters in a wide variety of forms and spoken words in a variety of voices and pronunciations. These are extremely difficult tasks for machines.

6. *No one knows much about these children's capacity to learn, but the limits may be far beyond what they now learn.*

Research indicates that a child's IQ can be raised significantly by exposing him to a stimulating environment. A few outstanding teachers have had remarkable success in teaching slow learners.

7. *If these children do not acquire abilities which make them employable, they become lifelong public charges.*

Since my only intensive contact with children in stage I is with my own two youngest children, I shall concentrate on the problems of bringing about the transitions from stage II to III and from III to IV. At stage II the child has acquired language. He can use a mudpie as a symbol for a pie. He judges by global perception. For example, if you place before him a set of vases, each containing a flower, and then take out the flowers and bunch them together, he says that now there are more vases than flowers. He is able to concentrate on only one aspect of a situation at a time. For example, the child will judge which of two cylinders contains more liquid on the basis of height or width, but not both.

He thinks egocentrically. He may know his own left hand, but he cannot tell which is the left hand of a child facing him. He thinks in terms of real actions, performed here and now. He has difficulty in anticipating the result of an action without performing it, or in reasoning "if I had done this yesterday, then . . ." or "if I should do this tomorrow, then. . . ." The child has difficulty in predicting the result of reversing an action or of combining two actions. He does not spontaneously imagine doing such operations, e.g., putting the flowers back into the vases. In stage III the child still thinks in terms of actions performed on real objects here and now. But he is able to anticipate the result of an action, to reverse an action mentally, to combine two actions mentally. He can think of several aspects of a situation simultaneously.

Piaget says that the child in this stage has formed mental "groupings" of concrete operations. Such a grouping is a system of imagined actions on real objects, in which each action can be reversed and any pair can be combined. An operation does not exist in isolation, but always as part of such a mental structure. The changes in thinking that must take place in order to pass from stage II to stage III are clear, but how can we facilitate this transition for a child who has already lagged behind, either because of low native ability or impoverished experience?

During stage IV, the child is able to perform mental operations on concrete operations, and to reason about general or hypothetical situations in addition to situations before him here and now. He can apply general principles. He can theorize. He can reason verbally or symbolically. He now possesses in his mental tool kit groupings of formal operations (mental operations) on concrete operations, which can be reversed and combined. This indicates the changes in thinking which we must bring about in order to help the child pass from stage III to stage IV. After methods of bringing about these transitions more effectively have been devised, materials will have to be created which will enable people of low mathematical competence to apply the methods successfully.

Some Suggested Approaches

To interpret the above summary of the results of research, we must emphasize that almost all the research to date is "status quo" research. Piaget and others have, in general, tried to find out how a child performs certain tasks, with as little interference with the child's thinking process as possible. While many people have been trying to draw inferences for education, many of these inferences are based on misconceptions. For example, some people have taken Piaget's stages and average age ranges as fixed, and infer that it is useless to try to teach certain topics before certain ages.

Most of the research needed in order to make sound applications to education has not been done, for the simple reason that, in the subculture of psychologists, experimental psychologists are high, whereas educational psychologists are low in the pecking order. Many of the most talented psychologists working on concept formation pride themselves on their purity. They are afraid to do experiments which involve teaching something to the child for fear of being confused with educational researchers!

This latter attitude also rests, in my opinion, on a misconception. Suppose I wish to find out what stage a child is in in the attainment of a concept, such as invariance of one-to-one correspondences under rearrangement of the members of the sets. If I set the child a task and observe how he performs, I do not know whether his response is due to his innate ability at that time or to the content of his past experience. I can find out better what he has the capacity to do if I try to teach him something as ingeniously as I can—so that I know that the child has been exposed to certain definite experiences, and then I can observe what the child learns. I can probe the child's mental structures and processes in a teaching-learning situation much more profoundly than if I observe him as he comes to me, with an unknown cultural background.

This type of research also has much more direct bearing on education than most existing psychological research. Research on the way children respond if you do not tamper with them is useful in identifying the psychological hurdles we must overcome. In education, however, we are concerned with what changes we can effect in the child's development through planned sequences of experiences.

I don't believe that psychologists will learn very deeply about learning until they team up with the cleverest teaching talent they can find. Let us now proceed to some specific problems. A few people, such as Smedslund, Wohlwill, and Dr. Muller-Willis and I (in collaboration), have investigated whether the child's development through stages II to III, with respect to particular concepts, can be accelerated. It appears that this can be done. Smedslund's results on the attainment of the concept of conservation of substance under deformation or subdivision indicate, however, that when this development is artificially accelerated, the mastery of the concept may be unstable.

For example, he was able to lead children to predict correctly whether two clay balls which balanced each other would still balance each other if he deformed or subdivided one. But if he concealed a bit of clay during the experiment in the palm of his hand, those who had not firmly attained the concept accepted their new observation as a curious fact, but those who had fully attained the concept accused him of cheating. It is not clear to what extent Smedslund's results could be improved with better teaching techniques. In our experiments, Dr. Muller-Willis and I were able to lead children to attain certain concepts of conservation about 1½ to 2 years before Piaget says they normally would. We hope to test our hypotheses more rigorously during the next year.

What does psychological research suggest regarding the strategy of passing from stage II to stage III? You want the child to reverse actions himself, observe the results, and state what he observes. If you want him to learn that a set of objects is invariant under rearrangement, you present him with a set of easily distinguishable marbles and ask him to identify them. "There is a red one, a green one, a yellow one, etc." You scatter them around the room, and ask him to put them back where they were before. You make the task easier for him if you present the objects to him in a pattern:

0 0 0
0 0 0

Then he looks for the red one and places it where it was before. He can see where there is a gap, and knows what to look for. He finds that no matter how you scatter the objects, he can always put them back.

You give him other sets of marbles, sets of other objects, presenting them in more, then less, obvious patterns. You begin to ask him to predict whether he can restore the set, and to test his prediction. Ultimately he knows in advance that they are the same marbles, no matter how you scatter them. He knows that he can put them back, without actually doing it.

When a child says, "There are more flowers than vases," or "There is less juice in this glass than in that one," I don't know for sure what he means by "more" or "less." I do not know what he associates his words with. If I want him to internalize his actions, I must give him the mental tools, the language, which will help him think. So I don't ask him whether there are more vases than flowers or flowers than vases. I start with dolls and hats, because it is hard to put more than one hat on a doll. "Here is a set of dolls and a set of hats. Put one hat on each doll. Are there any hats left over? Aha, then there are more hats than dolls. Let us scramble the dolls and hats again. Now put a hat on each doll again. Now what is left over? Some hats again? Amazing, isn't it! There are still more hats than dolls."

I attach the words "more," "less," and "just as many" to the action of matching one-to-one, and to the specific description of the result of the action. I act on the objects and ask the child to reverse my action, then ask him to describe the results. Then I have him act, reverse, and describe. Then he acts, predicts, reverses, and describes. I make sure that he uses the words to refer to the results of the actions performed. Similarly, "one quart" means the amount of liquid in this particular cylinder, when the level is at this particular point. I don't ask, after pouring into a vessel of a different shape, "Is there more or less now?" I ask, "Pour it back. Does it come higher or lower than before? That's strange, it came to the same height. There is just as much now as before. Try the experiment with these jars and bottles."

We try deliberately and consistently to associate the language with the actions and the observed results. First we have the child act, observe, describe; then he must predict, act, observe, describe. He is asked to reverse and combine actions, to observe and record results, to compare, and so on. Instead of giving the child a task in a rather mature situation, as psychologists do in testing the child's attainment, we give him the task at first in a very simple situation. We try to make the setting as foolproof as possible, so as to maximize the probability of success. We increase the difficulty only gradually, and present the mature setting only after the child has successfully performed the task in simpler settings.

Let me now turn to the problem of the transition from stage III to stage IV. In order to think about a proposition and to understand what it implies, you must be able to state it. Therefore, the attainment of stage IV is inseparable from the acquisition of the linguistic tools of thought. (By "language" I mean mathematical as well as spoken natural language.) In our haste to impart to the child the socially agreed upon signs (both words and mathematical symbols), we overlook the serious difficulties which our hodgepodge of historical accidents creates for children. The trouble is that our signs and the rules for their manipulation have no simple relation to the things they stand for or to the operations we perform on them.

Consider, for example, the way we symbolize the addition of 1 to a positive integer. We represent a positive integer by a string of symbols (0, 1, 2, 3, 4, 5, 6, 7, 8, 9). How do we represent the operation of adding 1?

The rules are:

change a final 0 to a 1 ($x0 + 1 = x1$)
change a final 1 to a 2 ($x1 + 1 = x2$)
change a final 8 to a 9 ($x8 + 1 = x9$)
if the string ends in a 9, add 1 to what goes before, then tack on a 0 ($x + 1 = y \rightarrow x9 + 1 = y0$)

These symbols and rules have no obvious relation to the operation of adding a new member to a set. Since most children are able to master these rules, either mechanically or with understanding, fairly early with present teaching, we take this learning process for granted. We do not realize how difficult a mental task it is to relate these symbols and their rules to the actual things.

In the work with slow learners, we may learn much that will be of great value in our curriculum development for the rest of the population. When we know that most children can learn to compute the sum of two positive integers in Hindu-Arabic-decimal notation by age 10, we may consider it risky to experiment with new notations in the first grade before we are sure that the transition will be painless. We may be bolder in our experimentation with low achievers, since the risk that we will interfere with their normal progress is less.

Remarks on Motivation

The students we are concerned with here cannot appreciate long-range rewards as easily as average students. The chance to drop out of school now, to take a job as a delivery boy tomorrow, to buy a car on time next week has a more immediately perceptible value than to struggle through 2 more years of school, to qualify for a more difficult job, to be able to support an unknown wife and family some 5 years from now. The rewards must be more obvious, immediate, and probable, the punishments less threatening and probable, and the values easier to conceive and perceive.

The most immediate reward is success, which is conducive to fun. The tasks must be broken down into pieces in which all children have a high probability of success. There must be fairly frequent fresh starts, varied enough so as not to become repetitious, so that failure and discouragement do not become cumulative. Up through the ninth grade the major emphasis can be on rewards of immediate recreational value. Even the mathematics involved in keeping score and computing batting averages makes the child competent to play with his peers, so that he need not seek less socially desirable avenues to self-assertion.

Even slow learners can obtain esthetic satisfaction from seeing the way relationships fall into patterns and structures. For example, in our 30 experimental classes per year in Minnesota since 1959 with the SMSG M-materials in grades 7–10, we found that the students were much more highly motivated than similar groups with conventional remedial or general mathematics courses. They turned in twice as much homework, participated much more in class discussions, and even engaged in extracurricular mathematical activities. Yet these SMSG materials make little reference to any practical applications of mathematics.

Still, having fun is only one of the important things a person can do with mathematics. It is also important for a person to learn that he can use mathematics in making a living, in managing a home, or understanding what politicians are asking him to vote on.

Many of the children we are talking about need specific motivation to prepare them to assume adult responsibilities. I would suggest that, beginning in grades 4–7, the children begin to see how mathematics is related to the world of work. They are beginning to think about growing up, and want to imitate grownups. During this period they should begin to use mathematics in shop, in home economics, in shopping, in play stores, in student government, and the like. The mathematics curriculum should begin to include more and more problems which arise in these contexts.

As the children approach the age when they are making vocational decisions, the situations in which they use mathematics should become more and more lifelike. In the shop the "carpenter's apprentice," as he makes real things, should be reading diagrams and blueprints, measuring, estimating and computing. The girls should begin to get experience with modern office equipment, including desk computers, punchcards, etc. It may be appropriate also for the girls to see the work women do in a modern electronics plant.

Materials written especially for these students must scatter its shots to lead into many mathematical topics. The exercises should also illustrate a variety of applications.

I strongly recommend that leaders from modern vocational education, training directors in industry, and education directors of labor unions be drawn into the work of planning the curriculum for these students and writing the materials. Arrangements should be made for experimentation with the materials in vocational education, work-study, and worker retraining programs. Provision for such curriculum research and development should be made in legislation for such programs, to be administered by the Department of Labor and the Department of Health, Education, and Welfare.

I have outlined a strenuous program of research and development which will require heavy investments of time, manpower, and money. I have indicated also the possibilities of tremendous payoffs. There is hope now, I believe, of great accomplishment once a good program is underway.

MATHEMATICS FOR LOW ACHIEVERS: RESPONSIBILITIES OF SCHOOL ADMINISTRATORS

by

George B. Brain*

In the years ahead, more mathematics, not less, must be taught to more students. Students aspiring to be mathematicians, physicists, engineers, scientists, astronomers, and the like must, of course, learn the most abstract mathematics. Few learned persons would question this, or that the average person needs an understanding of the fundamental processes of arithmetic. But what of those persons who test just below average in intellectual ability? Do they not require a mathematics curriculum geared to their needs and demonstrated learning ability? In Baltimore we think they do.

Years ago, the mathematics curriculum emphasized preparation for college, as it still does in most schools today. Schools must continue to offer college-preparatory mathematics courses, in greater variety and with better teaching than ever before, but they must also offer programs for those of low ability; for a knowledge of mathematics is essential for each and every student if he is to function effectively in today's world. Except for a few modest attempts in demonstration programs, however, low achievers have been too often neglected despite their need.

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Philosophical Considerations

Research into the psychology of learning indicates that students of low ability can learn much more mathematics than they are learning at present. Interestingly, there is evidence that students of low ability can learn mathematical concepts and processes at ages 16 to 18 that they were not able to learn when they were younger. This evidence holds great importance for curriculum and learning experiences. Even though slow learners have a short retention span, they *can* learn the sound, practical, everyday mathematics that they need in order to manage their personal affairs wisely, and to qualify for jobs and perform them efficiently.

Need for Orientation of Staff and Community

Too often, some members of the teaching staff and many good citizens in the community look upon mathematics as the "Queen of the Sciences." They regard with disdain the so-called watering down of mathematics courses for low achievers. To counteract this attitude, an effective orientation program, explaining the philosophy and objectives of such a course, is needed for both the teaching staff and the ordinary citizen. Otherwise, a mathematics program for low achievers is doomed to mediocrity at best and, more likely, to failure.

Too seldom have schools capitalized on the talents of citizens in the community. particularly in the great cities. Yet the basic mathematics program stands a better chance of getting proper recognition if some citizens are directly involved in the design of the curriculum and if they are acquainted with requirements for staff training and for the instructional program. Not only can they often contribute sound ideas, but they can also interpret the program to others in the community, thus arousing public understanding and support. Furthermore, citizen involvement has an important feedback feature: it helps keep the subject matter attuned to the needs of the business community.

In addition to giving teachers a proper understanding of the importance of the basic mathematics program, the superintendent and his staff should, by both word and deed, stress the importance of the work accomplished by teachers of low-achieving students. The community, too, can play an important role in recognizing the special service of this group of teachers.

Objectives of the Basic Mathematics Program

The program developed should be broad in scope, flexible in nature, and adaptable in application to each student's needs. Here are the objectives set forth for courses for low achievers in Baltimore:

(1) To increase the student's skill in fundamental operations of arithmetic;

(2) To develop his ability to meet mathematical situations effectively in the home, school, business, and commuity;

(3) To enable him to use simple formulas, equations, ratios, and proportions;

(4) To teach him essential aspects of informal geometry as related to practical real-life situations;

(5) To increase his understanding of direct measurement;

(6) To teach him elementary techniques of problem-solving;

(7) To help him develop a vocabulary rich enough to understand and express mathematical ideas in daily life;

(8) To develop his ability to think through a quantitative situation, make sound judgments about it, and appraise the reasonableness of his judgments;

(9) To help him become a more intelligent, more critical consumer;

(10) To develop his appreciation of the role that mathematics plays in making advances in the modern world;

(11) To prepare him adequately for further courses in mathematics.

In accordance with these objectives, criteria for student admission, selection of staff, training of staff, teaching materials, program evaluation, pupil promotion, remedial work, and class size must be determined under the leadership of the school administrator.

Pupil selection

Pupil selection criteria should be clearly and concisely stated. Under no circumstances should this program be considered a dumping ground for students with behavioral problems. Rather, the students selected should be those who, despite being low achievers, have a desire to learn, and, even more important, those whose parents want them to learn also.

The low achievers can best be characterized as students having an intellectual potential at the low-average level, who are generally about one or more years below grade in arithmetic and one or more years retarded in reading. These characteristics should serve as guidelines for admitting pupils to a basic mathematics program, but not as inflexible criteria, to be rigidly applied.

Teacher selection and training

Teachers for these programs should show the same personal characteristics that superior teachers for any other school program do: a fondness for young people, emotional maturity, physical stamina, a broad fund of knowledge and curiosity for more, and a sense of humor. In working with low achievers, the teachers should show an interest in the pupils as individuals; apply rights and rules equally to all; communicate with them on their level of understanding; individualize the instruction as much as humanly possible; stimulate the pupils to think, evaluate information, and substantiate conclusions; and try new and different teaching techniques and routines. In working with their colleagues, these teachers should cooperate in teamwork with other teachers and administrators, add continually to their knowledge of subject matter and teacher techniques, exhibit responsibility and loyalty to teaching as a profession, and welcome professional evaluation of their teaching performance.

If prospective teachers are to be imbued with these characteristics, they must undergo intensive professional training. Local colleges and school systems should make provisions for appropriate course work for teachers of basic mathematics programs. School administrators should not overlook inservice training activities—workshops, interschool visits, and demonstration teaching. Supervisors can also play an important role in training teachers for basic mathematics programs.

Continuous program evaluation

Today's job structure is unlike yesterday's, and tomorrow's will be unlike today's. Technology will have rendered obsolete many present work procedures and much of the knowledge needed for effective functioning in today's society. For this reason, administrators must make provision for continuous evaluation of content and teaching methods, so that the students will have been taught the skills they need to enter the world of work at any given period.

Summary

The role of the administrator in providing a basic mathematics program for low achievers should be as follows:

> He must emphasize to staff and community alike the need for the program.
>
> He must initiate and provide for an effective staff and community orientation program.
>
> He must give to teachers of the slow learners proper recognition, stressing the importance of their work.
>
> He should involve community leaders as much as practicable and make effective use of the special talents and resources of various citizens.
>
> Within budgetary limitations, he should arrange a workable class size that permits teachers to devote the necessary attention to individual differences and remedial instruction.
>
> He must enable the teachers to develop and use new materials for low achievers. Improvisation and innovation must be encouraged.
>
> He must make practical, realistic inservice programs available for teachers working with low-achieving students.

Students of low ability *can* learn mathematics; they *can* perform useful work; they *can* make an effective contribution to society. It is up to the school administrator to see that his school system has an effective basic mathematics curriculum to enable these pupils to realize their potential.

HOW CAN BUSINESS AND INDUSTRY COOPERATE WITH SCHOOLS ON THE PROBLEMS OF LOW ACHIEVERS IN MATHEMATICS?

by

George A. Rietz*

I am delighted that the word "cooperate" appears in the title of this address. It matches the philosophy of most representatives of industry who work in the educational field, which is to work with and through educational institutions. It also suggests that schools should look upon other institutions in the community as allies vitally concerned in the upbringing of our children. The American Association of School Administrators has stated it in this way: "The school that uses its community as a teaching and learning laboratory, is using live ammunition." I assume my role is to spark a discussion including *what* kinds of help can be expected from business and industry, *why* such help can produce results, and *how* a school can get such help, or "cooperation."

Perhaps we should first take an oversimplified look at any employer, in any community. He is interested in hiring people with ability, good character, and motivation to help him succeed in his business venture. He wants to match each employee's interests and abilities to a job on which the employee can make the greatest contribution while deriving the greatest personal satisfactions. The employer must compete with similar merchants, banks, or manufacturing operations in the community, the region, or perhaps internationally. Thus, he wants people who will get things done so that he can serve the customers' desires at competitive costs. He interviews a number of applicants, selecting the ones who seem to offer the greatest possibility of top performance. He also provides them with some training on the job so they can become productive most quickly.

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Particularly in large organizations, the job on which an employee starts may bear little resemblance to positions he will hold 5, 10, and 25 years later. The alert employer is constantly looking for evidence of ability and desire to handle greater responsibility, and encourages the employee to continue his education and training throughout life. Knowing that few work to anywhere near the level of their ability and that motivation can make up for a considerable lack of brilliance, industry generally is less inclined than schools to "write off" or permanently classify an individual on the basis of "tests." At the same time, employers know of the strong correlation between success on the job and academic grades. You may be amused at the answer one of our executives presumably is said to have given to the question, "How hard do you expect an employee to work?" His reply was, "Not so hard that he will be as tired on Friday night when he leaves the job as when he returns to it on Monday morning."

Dr. Hansen, Superintendent of Schools in the District of Columbia, has said, "Educational effort is primarily an expression of hope on the part of the student." Isn't our first question this: What help can local representatives of business and industry give teachers in developing a greater degree of hope in the students? That help can have three important objectives:

> To show how proficiency in mathematics (and other subjects) will help each student to achieve success in his future business and personal life,
>
> To give a more realistic picture of employment, so that students are motivated to prepare adequately for exciting and rewarding careers rather than for "a life sentence of drudgery," and
>
> To understand that students lacking a good educational foundation will experience difficulty—first, in getting a desirable job, and, thereafter, with necessary continuing education and training programs on the job.

Now let's take a look at such things as employment, success on the job, and the changing pattern of jobs, with inferences about the importance of mathematics.

Twenty years ago, about 25 percent of the average large plant's employees were employed in unskilled jobs, whereas only about 15 percent are today. Another interesting statement is that approximately one-half of the employees with my company now work on products that did not exist before World War II. The statement by Lawrence Rogin, Director of Education, AFL-CIO, should be read to your students: "The best assurance of future job security is a good general education. Tomorrow's worker will need to read with understanding, write clearly and figure accurately, at the very least. Such basic knowledge and skills will give him the flexibility he needs to learn new techniques and adapt himself to new jobs. The skilled worker will need more mathematics and science than he has now."

Any employer will give his own summary of reasons why some people are not hired. He may state that he places about 75 percent of his appraisal on character traits, but usually he will have first assured himself that the applicant has special skills required for the job.

Employers should talk with students and teachers about the realities of the world of work. For example, the following is extracted from a talk given by one of our plant personnel to a community meeting on high school dropouts on "Why Employers Judge Against the Dropout as an Industrial Risk":

"He is probably unable to perform basic arithmetic or deal effectively with physical measurements. . . . He probably lacks self-discipline, pride, ambition, or drive. . . . There is cause for concern about placing marginal people in charge of expensive, complicated machinery. . . . He probably has a very low, or else twisted sense of economic responsibility. . . . He hasn't been able to grasp the fact that he will be rewarded in proportion to his contribution . . . The worker must be flexible in his attitude and willingness to do different kinds of work, frequently on short notice (to meet changing customer demands). The dropout has proven himself unable to adapt to the requirements of the school and therefore is a bad industrial risk. . . . If it were a well-known fact that high school graduation is practically a requirement of employment, then there might be fewer optimistic youngsters leaving school in full confidence of finding a job. There is no point in being a Pollyanna about this. The possibility of getting a job and thereby becoming an 'adult' is a great driving force which pries many young people out of the classroom. If he *knew* he would not be hired before graduating, a great part of the temptation might be removed. And if this *is* a fact of life, he may as well face it."

At least two findings reported in studies of this subject bear on this discussion of dropouts. First, a smaller percentage of dropouts hold after-school and Saturday jobs than did those who graduated. I don't know whether students learned of the need for more education from their jobs, or whether some jobs made school appear more attractive. Second, about three out of every four dropouts are poor readers. Poor reading ability relates to low achievement in mathematics, in most other subjects, and even to failure in college. Mathematics teachers have been "campaigning" against teachers in the elementary grades who "pass on" their fear of mathematics to their students. Is there a need to campaign for topnotch reading instruction in your elementary schools?

Going back to the world of work, our student should know that good intentions are not adequate preparation if he hopes to help solve his own or society's problems. It is important, however, that we avoid pressuring students vis-à-vis the supposed relative status of jobs. There is a tremendous need to instill a feeling of pride in accomplishment and in the dignity of constructive work. John Gardner's statement will serve as an appropriate text on this point: "An excellent plumber is infinitely more admirable than an incompetent philosopher. The society which scorns excellent plumbing because plumbing is a humble activity, and tolerates shoddiness in philosophy because it is an exalted activity, will have neither good plumbing nor good philosophy. Neither its pipes nor its theories will hold water."

We have seen how lack of a good educational foundation handicaps the student in landing a good job. The table below shows the relative performance of employees on upgrading training programs for highly skilled mechanical jobs in one factory.

Grades attained in on-the-job training courses

Grade	High school graduates	Dropouts
	Percent	Percent
A	36	17
B	46	35
C	18	44
D	0	4

Is it difficult to understand why employers repeatedly state, "If the school does the educating, we can do the training and re-training"? Continued education and training are an ever-increasing fact of life, and can be an enjoyable key to continued success. At Crotonville, N.Y., where my office is located, thousands of employees, including vice presidents, and department, section, and functional managers, leave their job for a few weeks or even a few months of study to keep up-to-date and to broaden their education. A wide range of courses are offered at practically every plant and office of the Company, as well as university courses offered on-campus or in-plant. One of our officers estimated that, at any moment, one of each eight employees is taking some educational or training course, at a total annual cost to the Company of about $50 million.

How can the kinds of information we have been discussing best be communicated to students? I have purposely indicated that it can best be done at the community level. Before treating community-level programs, I would like to refer briefly to the help available from the headquarters of large companies or industrial associations. The large number of students involved prompts a search for what is often called, "the multiplier factor." Most activities are either intended to help the teacher upgrade his ability to teach, or to place at his disposal teaching aids for use in the classroom. Perhaps you know that 2,500 mathematics and science teachers participated in our 6-week Summer Graduate Fellowship program at seven universities between 1945 and 1959. Their performance in the classrooms confirmed the soundness of that program. Institutes with NSF support have become so generally available that our secondary school Summer Fellowship offerings have been in the guidance and economics fields, since 1959 and 1960, respectively. Teaching aids are usually bulletins, posters, or films. In 1953, we made available to mathematics teachers two bulletins: "Why Study Math?" and "Math at General Electric." Teachers requested nearly 3,000,000 copies in classroom quantities. Since 1954, "Why Study Math?" has been available in the bulletin "Three Why's" and more than 6,000,000 copies of that bulletin have been requested by teachers. About 40,000 teachers receive our bulletin board posters during the school year. We hope to offer again a "House of Magic" type of demonstration for schools. Perhaps you see our educational ads in student and teacher magazines. Also, we consult with, advise, and help more than 150 local Company branches in their work with local schools.

In any community, business representatives know of all services available from their headquarters, plus many other things they alone can provide. Until we made a survey, we were not aware of many of the cooperative activities with local schools carried on by many of our plants, and summarized in the publication "Youth and Education Programs." Activities designed to communicate information to students usually include talks at schools or plants; visits to business; cooperative work-study programs; inservice programs for teachers or students; making consultants available to individual students or teachers; mathematics clubs; contests; newspaper; radio and television; and, of course, films, bulletins, posters, and demonstrations as previously mentioned. We must look upon each type of activity primarily as another way to communicate the desired information.

How can the school get cooperation from local business people? There are two important reasons why the teacher (with consent of the school administration) must take the initiative in starting any cooperative venture. First, organizations and individuals in the community will be cooperating with the teacher in accomplishing objectives which are best known to the teacher. He must identify, select, and develop plans to utilize the many facilities within the framework of the school system. Second, in most communities, the suspicion that anyone who offers assistance to local schools has an ax to grind is so deeply imbedded that approaches by outsiders have often been met coolly or with rejection. For example, one of my associates cites a manager, new to the community, who went to the school indicating an interest in their programs, and offering assistance either as an individual or through his Company connection. A few days after having accorded him a cool reception, the head of the school system called, essentially asking, "I have been doing a lot of thinking about our conversation, and I just can't figure it out. Please level with me. Just what is it you want?" Let me hasten to add that the educator will rarely receive anything but a warm reception and a willingness to participate in any reasonable request. The program of cooperation and assistance must and can then be carried forward with full mutual understanding and respect.

The usual formal contact with a large organization might best be with the manager of community relations, or personnel or employee relations depending on its organization. In some instances, the education committee of the local Chamber of Commerce or other business association is the preferred contact. Programs should eventually involve qualified individuals as professional or working people in the community and others as representatives of many different companies. Remember that direct communication regarding the local employment situation is most applicable, and can bring new meaning and excitement to what may have been dull and theoretical to some students. Information on such things as observed shortages in student preparation, preparation required for various types of work, use of mathematics in business, opportunities for women, cooperative work-study programs, and hiring practices and standards are available in all but the smallest communities. Teachers from small towns usually can get much of the same help through cooperative programs which are conducted so as to utilize the facilities of business and industry located in the region.

Perhaps a few final words of encouragement and advice might be added. Most adults in the community will be pleased to have an opportunity to help students—perhaps it is based on their parental instincts. To do a good job requires hard work on their part. The teacher must plan carefully and thoroughly, state the objectives clearly, and insist on high standards of performance for every participant—students will respect him for it. If he selects the proper local business representatives and insists that they do their "homework" to attain the objectives of the cooperative venture, the students will gain other insights and stimulation from contacts with such representatives. Each of these representatives will undoubtedly reflect, though he does not put it in words, the fact that he finds his work challenging, important, and rewarding. The students shouldn't be exposed only to top people. At all levels, they should observe directly the respect and dignity of productive jobs as reflected by successful workers.

The "hope" of which Dr. Hansen spoke must be focused far beyond the end of this marking period, even beyond the end of high school. Mr. Kettering, Vice President of General Motors, often said, "We should think more about our future, that is where we will spend the rest of our lives." There are other stimulating ideas and facts about the future which will contribute to student hope and motivation. I wonder how many youngsters look forward to entering the work force with fear brought on by misinformation received from newspaper headlines, speeches, and books. Does the typical student look forward to a job with optimism and enthusiasm, knowing of the challenging career opportunities, or does he look forward with frustration and dread to entering the world of work? Shouldn't he be made aware of the freedom of choice he enjoys under our system, and that many graduates even "invent" entirely new jobs because they find that consumers will buy products or services they feel are worth the cost? Does he know that employers are always looking for people with ability who will get things done that may assure success in their business ventures?

Much of the student frustration and pessimism about the future could be alleviated considerably if students were also given a better understanding of such things as automation and unemployment. Hopefully, the findings of university research on automation—further mechanization—will some day get as much attention as the scare headlines and stories. For example, how generally known are the research findings by the University of Chicago and Cornell University? Briefly stated, they are, respectively, that automation creates employment and lessens the total number of unemployed, and that manufacturing turnover requires 10 times as many new employees each year as are displaced by all labor-saving devices.

My assigned topic—possible education-industry cooperation on the problem of low achievers—suggests the curative, remedial, or corrective approach. I feel sure that you all agree that well-planned and executed cooperative programs can help produce desired improvements. However, I cannot close my remarks without pointing to the probability of keeping many students from ever becoming low achievers. Hopefully, schools will recognize that the "live ammunition" available in the community should be "harnessed" to the benefit of all students. Several types of community activities—some once a year, others continuing through the school year—can help solve the problem being discussed at this conference not only in corrective but also in preventive directions. Many school systems can point to such accomplishments. Most adults in any community will be delighted to cooperate. Will teachers take the initiative?

PART II

CURRENT PROMISING PRACTICES

46/47

BALTIMORE'S BASIC PROGRAM IN SECONDARY MATHEMATICS

by

William J. Gerardi*

In September 1958, the Secondary Mathematics Department instituted a basic mathematics program specifically designed for students in the lowest 30 percent. The program began in the seventh grade in 1958 and has continued each succeeding year in the next grade level. By September 1963, the original group was enrolled in a 12th-grade basic mathematics course.

Student Selection

Students were recommended for the program by the elementary school principals, and final selection and assignment to homogeneously grouped classes of 25 to 30 were made by the receiving junior high principal. The criteria used in selecting students were as follows:

An IQ between 80 and 90;

A fourth-grade level of achievement in reading;

A fourth-grade level of achievement in arithmetic;

A record of poor achievement in mathematics;

At least seven years spent in the elementary grades beyond kindergarten;

A minimum of one year spent in the sixth grade;

A chronological age of at least 13 years, 8 months, on August 31.

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*Mr. Gerardi is supervisor of secondary mathematics in the Baltimore City Public Schools. Since 1958 he has been actively engaged in the development of a sequential mathematics program for low achievers in Grades 7–12. He is currently developing a pilot program which will make use of audiotronics devices and a multisensory approach to the teaching of underachievers.

Teacher Selection

Teachers, insofar as possible, were selected on the basis of the following demonstrated characteristics believed to be effective with slow learners:

> A good knowledge of secondary mathematics and its applications;
>
> A good knowledge of the characteristics of the slow learner;
>
> An enthusiastic, encouraging approach with a missionary spirit and a deep devotion to teaching;
>
> A good sense of humor, infinite patience, and a talent for ingenious presentation of material in a concrete, meaningful, interesting manner;
>
> A realistic and flexible standard of grading; and
>
> An attitude of empathy for the slow learner.

The teachers met as a group and were given some orientation. They were made to feel that they were selected to do an important job, one which required them to give their very best. They were told that they would play a key role in evolving and evaluating a realistic and meaningful program for an important segment of our society. The students had to be given the very best mathematics education possible if they were to become responsible, productive citizens in a rapidly changing, technologically oriented society.

Junior High School Basic Mathematics Program

We believe that the basic mathematics program for the 7th and 8th grades should include essentially the same topics as the regular program. Transfer to the regular program should be possible at this point for those who demonstrate through achievement that they have the ability. The 9th-grade program should parallel rather closely the regular general mathematics program in order to provide a spiral continuation of the 7th- and 8th-grade content. Remedial practice for specific difficulties and applications which tend to reinforce mastery of basic principles are stressed.

Throughout the entire program differentiation is made in levels of learning, depth, and scope. As compared to the regular program, the amount of concrete background is enlarged and the rate of presenting new material is curtailed. Subject matter within a topic is selected so that pupils can achieve on their level. Each pupil is expected to work up to his capacity and his work is expected to be neat and accurate. The Los Angeles Diagnostic Tests are given to 7th-grade pupils. Remedial instruction in arithmetic, based on analysis of the test results, is then planned for individual pupils. Brief, daily written and oral drills are designed to provide necessary remedial work, reinforcement of basic skills, and maintenance of important concepts.

Content—Grades 7, 8, 9

Although elementary arithmetic has not been mastered by pupils who enter the 7th-grade basic course, the students look forward to some new mathematics and not the same old arithmetic. The first unit in each of the junior high school years is therefore designed to present some new mathematics.

Grade 7

(5 periods per week)

Geometric Forms
Linear Measurement
Number System
Graphs
Measurement of Angles
Triangles
Areas of Plane Figures
Applications of Percent

Grade 8

(5 periods per week)

Working With the Decimal System
Percentage
Measurement
Applications of Percent
Circles
Measurement of Solids
Evaluation of Formulas
Equations and Problems
Directed Numbers

Grade 9

(5 periods per week)

Equations and Formulas
Directed Numbers
Graphic Representation
Constructions
The Right Triangle
Ratio and Proportion
Indirect Measurement
Applications of Percent

Senior High School Basic Mathematics Program

We believe that the 3-year basic mathematics sequence in high school should be an integrated study of arithmetic, algebra, and geometry. The unit on budgeting, for example, should involve the fundamental operations of arithmetic, an algebraic solution to percentage problems, and some informal geometry involving circles, central angles, and sectors. Algebra and geometry should be introduced only when the need arises or when their use would simplify or clarify a skill, concept, or application to be taught. We feel that the mathematics that is taught should be related to the probable needs in the lives of these pupils.

The presentation of the subject matter should stress key ideas and basic skills; it should also be varied, interesting, and functional. It should be clearly related to the present environment and should rarely surpass the range of the pupils' comprehension and experience. It should prepare the pupils to handle effectively the mathematical problems and experiences they will probably meet in later life. Lastly, it should be correlated as closely as possible with other subjects.

Content of Grades 10, 11, 12

Grade 10

(5 periods per week)

- Earning Money
- Budgeting
- Buying Wisely
- Installment Buying
- Home and Job Mathematics
- Borrowing Money
- Taxation
- Insurance
- Banking and Investments

Grade 11

(4 periods per week)

- The Number System
- Number and Operation
- Numbers in Measurement
- Rational Numbers
- Numbers in Percent
- Angles and Polygons
- Equations
- Perimeters and Areas
- Surfaces and Volumes
- Ratio and Proportion
- Indirect Measurement
- Financial Transactions

Grade 12

(3 periods per week)

- Slide Rule and Computer Mathematics
- Personal Finance
- Buying and Owning an Automobile
- Renting and/or Buying a Home
- Income Tax
- Industrial and Business Applications
- Social Security and Insurance
- Statistics and Probability

Teacher Training

The key to a successful mathematics program for the slow learner is a teacher who understands that slow learners are teachable, and cares enough to do his very best in teaching them. It is, therefore, important to provide teachers of basic mathematics classes with an orientation to the program, inservice courses, and many opportunities for intervisitation. Since very few colleges offer educational methods courses specifically designed for teachers who will work with slow learners in mathematics, we have attempted to provide some inservice instruction in this area. All new teachers are required to attend monthly meetings during their first 2 years of service. Many of these meetings deal with the following topics:

Characteristics of the Slow Learner
Motivating the Slow Learner
Planning for the Slow Learner
Evaluating the Achievement of the Slow Learner
Managing a Class of Slow Learners
Teacher Attitude and the Slow Learner

In addition, inservice workshops have been offered on Arithmetic in the Secondary Schools, Laboratory Mathematics, and Growth of Mathematical Ideas.

Evaluation of the Basic Program

Our program is now in its sixth year. We have many basic mathematics classes in each of the grades 7–12. As shown by the Stanford Achievement Test, the Snader General Mathematics Test, and individually designed departmental tests, the students enrolled in these classes have not only maintained, but have improved, their mathematical competencies. In addition, since these students have been grouped homogeneously and given a program designed for them, mathematics has had a greater holding power for them.

THE MATHEMATICS PROGRAM FOR THE LOW ACHIEVER IN THE FORT WORTH PUBLIC SCHOOLS

by

Jim Bezdek*

In the Fort Worth public schools, the program for the low achiever is an integral part of the total mathematics program. Mathematics is a required subject for all pupils through grade 8. Beginning with grade 9, the pupil must successfully complete 2 years of mathematics to satisfy high school graduation requirements. Three plans of instruction are available:

Type of plan	Grade 7	Grade 8	Grade 9	Grade 10	Grade 11	Grade 12
1. Minimum Plan	Arith 7	Arith 8	Related Math 1&2	Related Math 3&4		Consumer Math 1&2
2. College Prep. Plan	Arith 7	Arith 8	Alg 1&2	Geom 1&2	Alg 3&4	Trig & Elem Analysis
3. Accelerated Plan	Arith 7	Arith 8; Alg 1	Alg 2&3	Geom 1&2	Alg 4 & Trig	Elem Analysis 1&2*

*Or 1 year of Analytic Geometry and Calculus by invitation and special approval.

The greatest number of pupils are enrolled in the standard College Preparatory Plan, which is in full operation in all schools. The Accelerated Plan is scheduled for full operation in all schools by 1968. The Minimum Plan is to be offered on a pilot basis to approximately 300 pupils at the 9th-grade level in September 1964, and then extended to the 10th grade in September 1965. If the pilot classes prove effective, the Minimum Plan will be in full operation in all schools at the 9th-grade level in September 1965, and in the 10th grade in September 1966.

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*Mr. Bezdek serves in the dual capacity of Assistant Director of Curriculum and Research and Consultant in Mathematics and Business Education in the Fort Worth, Tex., public schools. He has provided leadership in recent local curriculum revisions in mathematics education and continues to be active in efforts to provide an adequate program for the low achiever.

The Minimum Plan is designed for the low achiever who needs additional work in the foundation of mathematics and/or who does not plan to continue his education beyond high school. The need for additional work is exhibited in several ways: (1) failing or weak passing marks in the arithmetic of grades 1–8, (2) extremely low scores on standardized achievement tests and educational ability tests, and (3) lack of desire to learn mathematics.

Local provisions for the low achiever are based on the assumption that he *can* learn basic mathematics, but slowly. The approach is to spread the equivalent of first-year algebra over a 2-year period and to include elements of geometry, trigonometry, and statistics. It is believed locally that this combination is really the "general mathematics" needed for effective citizenship in today's world.

The low achiever must be convinced that his study is important and that he is respected, if the instruction is to be effective. To accomplish this, the plan must have a recognized place in the total instructional program. The Minimum Plan described above meets this criterion. If the low achiever rectifies his deficiencies sufficiently in grades 9 and 10, he may then begin work in grade 11 in either Geometry 1 or Algebra 3 and still complete a satisfactory college-preparatory program. This feature gives the Minimum Plan flexibility and status.

The two basic courses in this plan at the 9th- and 10th-grade levels have been designated Related Mathematics. This was done to avoid using the traditional designation General Mathematics, which has fallen into disrepute locally with pupils, parents, and teachers. In an attempt to give the plan more status, it is designated the Minimum Plan for meeting graduation requirements rather than Non-College Bound Plan or Terminal Plan.

Providing for the Low Achiever at the 7th-Grade Level

The Minimum Plan, which begins at present in grade 9, does not take care of the needs of the low achiever at the 7th- and 8th-grade levels. These years are most critical. Obviously, if the pupil is deficient in arithmetic at the 7th-grade level and nothing is done about it, he will also be deficient in the 8th grade, and totally lost if he ever succeeds in reaching the 9th grade. These pupils become candidates for dropping out of school and/or for renouncing further study of mathematics. The problem is compounded the longer one waits in the education of the child to rectify his deficiencies. There must be a planned approach to raise the mathematical level of these pupils *before* they reach the 9th grade.

In 1963–64 the Fort Worth schools set up an experiment involving three classes of 7th-graders who, upon entering the 7th grade, had an average deficiency of approximately 2 years in both reading and arithmetic. The pupils in these classes were matched on a one-to-one basis according to IQ, age, sex, and evidence of reading and arithmetic difficulty. One class is using the regular textbook as in the past; a second class is using programed instructional materials; and a third class is using individualized instructional materials designed to help each of them overcome weaknesses in basic understandings and skills. The same instructor is teaching the three classes. It is hoped that the evaluation of this experiment at the end of the current year will shed light on the effectiveness of special approaches and give direction to other local efforts along these lines.

In a study of local pupil withdrawals during the 1962–63 school year in grades 7–9, lack of motivation and poor achievement were cited as reasons by 64 percent of the pupils. The survey further indicated that 75 percent of the dropouts had less than average educational ability.

An experimental program specifically designed to meet the increasing dropout problem will begin next September in two Fort Worth junior high schools. One of the basic objectives of the program will be to whet the desire for self-improvement in each pupil. By providing an educational program specially designed for the low achiever—a program that he understands and accepts—we hope to keep the child in school as long as possible. In addition, the program will seek to improve the pupils' ability in reading, writing, spelling, oral expression, and arithmetic to the maximum of his potential, at the same time seeking to develop good citizenship and love of country. The ultimate goal is to return the child to the regular classroom as quickly as possible.

Pupils selected for the pilot classes will be those who are at least 13 years of age upon their entrance into grade 7, who are 2 or more years retarded in reading and mathematics, who have a record of continuous low achievement and irregular attendance, and who have a record of emotional and social maladjustment. The program will include two consecutive hours in the basic skills of reading, spelling, and grammar. The project teacher will instruct these language arts sessions. Mathematics will be taught at the group level by a teacher sympathetic to the problems of the group; and, for the remainder of the day, pupils will be mixed with regular classes. In this program, pupils are scheduled to take 4 years to complete the 3 years of work usually assigned to the junior high schools in grades 7–9.

Level I, First Year

Language Arts	2 hours.
Mathematics	1 hour.
Texas History	1 hour.
Physical Education	1 hour.

Level II, Second Year

Language Arts	2 hours.
Mathematics	1 hour.
American History	1 hour.
Physical Education	1 hour.
Elective	1 hour.

Level III, Third Year

Language Arts	2 hours.
Mathematics	1 hour.
Physical Education	1 hour.
Electives	2 hours.

Level IV, Fourth Year (Credit Given)

English	Physical Education.
Related Mathematics 1 and 2	Elective.
Science	Remedial work or supervised study.

The "M" Materials of SMSG

by

Max A. Sobel*

In the summer of 1959 the SMSG (School Mathematics Study Group) formed a panel to study the mathematics program for noncollege bound students of average and below-average ability. This group, later named the Panel on Underdeveloped Mathematical Talent, undertook to make recommendations for the group of pupils in the 25th to 75th percentile ability—the so-called middle, or "M" group of students, who are of average or slightly below-average ability. Summer writing teams were directed to prepare materials at two levels: (a) grades 7 and 8, and (b) grade 9 algebra.

The materials developed were used to test the hypothesis that students of average and below-average ability can learn the kind of mathematics presented in standard SMSG texts, provided they are permitted to proceed at their own pace through a presentation geared to their own level of ability.

Three summers of writing and two years of classroom experimentation finally produced the following materials, now available from Yale University Press: (a) *Introduction to Secondary School Mathematics* (IS), and (b) *Introduction to Algebra* (IA).

Each of these courses consists of two volumes and each is intended as a 2-year sequence. IS covers much of the same material that appears in the SMSG text "Mathematics for Junior High Schools," Volume I, together with selected portions from Volume II of this series. IA covers essentially the same mathematical content as in the SMSG text "First Course in Algebra." Some of the basic differences are that the IS and IA texts were prepared with the following guides in mind:

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> To adjust the reading level to the comprehension of the less gifted pupil;
>
> To shorten chapters, as well as sections within chapters;
>
> To introduce new concepts through concrete examples;
>
> To provide numerous illustrative examples, as well as more detailed explanations;
>
> To include more simple exercises and drill materials;
>
> To provide numerous summaries, reviews, and cumulative sets of problems;
>
> To pay greater attention to basic computational skills;
>
> To reduce the number of abstractions; and
>
> To increase student participation in the development of ideas through discovery techniques.

To date, the subjective impressions gathered by various members of the panel, as well as those communicated by the many teachers who used these materials experimentally, indicate their effectiveness for the "M" group. These texts are *not* offered as appropriate content for the very slow, noncollege bound student.

IA is especially well suited for that large middle group of students who are all too often lost in the conventional programs. Normally these are the youngsters who either cannot pass a first course in algebra, or else who are placed in a general mathematics class in which they fail to achieve success or satisfaction. Nevertheless, many of these same students master a first course in algebra if it is given at a moderate pace over a 2-year period.

IS was designed for junior high school youth whose mathematical talent is underdeveloped. In this group also, there will be some who possess undiscovered mathematical talent. As stated in the preface to teachers, "It is hoped also that an understanding of fundamental concepts can be built for those whose progress in mathematics has been blocked or hampered through rote learning or through an inappropriate curriculum. We hope that appropriate mathematics, suitably taught, will awaken interest in pupils whose progress in traditional courses seemed hopeless. The discovery and nurture of heretofore unidentified capacity for learning mathematics is one of the main purposes of this book."

The 2-year sequence of IS texts was thus designed for students of average and slightly below-average ability in grades 7 and 8. At Montclair State College, however, which was a center for experimentation with these materials, they proved to be most appropriate for very slow students in senior high school general mathematics classes, and they are currently being used in this capacity by many local schools. For this group, the IS texts present a fresh and exciting approach, offering a body of subject matter that seems to interest the very slow student while at the same time providing him with the necessary work on fundamental skills. Furthermore, for many students of below-average ability, these texts have high potential value as a prealgebra course at the 9th-grade level. Thus, one may consider a 4-year sequence consisting of IS in grades 9 and 10, and IA in grades 11 and 12.

SMSG is now in the midst of a careful study and evaluation of the "M" materials with approximately 40 classes at the 7th-grade level and an equal number at the 9th-grade level. Although final results will not be available for at least another year, the panel on the underdeveloped mathematically talented is nevertheless optimistic that this study will indicate the effectiveness of these texts for students of average and below-average ability in mathematics. As stated in the preface to teachers, the panel hopes that with this approach, "we shall be successful in attracting and retaining increased numbers of pupils for continued study of mathematics."

PREPARING THE MNCB TEXT "EXPERIENCES IN MATHEMATICAL DISCOVERY"

by

Oscar F. Schaaf*

In 1962 the National Council of Teachers of Mathematics appointed its Committee on Mathematics for the Noncollege Bound (MNCB) Student, and assigned to it the task of writing a general mathematics text. During the summer of 1963 the Committee wrote *Experiences in Mathematical Discovery,* which is being tried out during the school year 1963–64 in 50 or 60 classrooms in various parts of the country.

Prior to the actual writing of the text, the Committee agreed on the following assumptions:

(1) There should be a mathematics course geared to the ability of any student enrolled in school. The course should be difficult enough to challenge each student, yet easy enough for him to experience success.

(2) Mathematics courses in the regular sequence of an up-to-date mathematics program in which sound pedagogical principles are practiced are of value to all students, but no student should be placed in a course until there is reasonable assurance that he will succeed in it if he applies himself. A student should be allowed to progress through the sequence as far as possible and at a rate commensurate with this ability. If the regular courses are con-

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sidered the "main track," then there should be "side tracks" along the way to serve motivational, vocational, and remedial needs. (This does not mean a rigid separation of students into classes for the college-bound and the noncollege bound.)

(3) In general, students who are deficient in mathematics should be given remedial instruction as soon as the deficiency becomes apparent.

(4) The School Mathematics Study Group series, *Introduction to Secondary School Mathematics,* Volumes 1 and 2, are suitable for 7th- and 8th-grade students in the 25th to 75th percentile range of mathematical achievement.

A primary concern of the Committee was to determine at what point "side track" courses are needed; they recommended giving priority to the production of sample materials to be used in courses for students in the following categories:

> Seventh-grade students below the 25th percentile of mathematical achievement;
>
> Ninth-grade students below the 25th percentile;
>
> Ninth-grade students in the 25th to 50th percentile range (the category for which *Experiences in Mathematical Discovery* was written);
>
> Low achievers (not failures) in algebra and geometry who wish to continue their study of mathematics;
>
> Eleventh- and twelfth-graders who do not demonstrate a reasonable mastery of basic mathematics; and
>
> Seniors who want a terminal, nonremedial course in consumer or practical mathematics.

If suitable texts were available for students in these categories, most secondary schools, the Committee believes, would be encouraged to develop better mathematics programs for them.

Since nearly all secondary schools have a general mathematics program, however, the Committee felt that this area should receive attention first. Courses in 9th-grade general mathematics have met with disfavor from both students and teachers. Probably one of the main reasons is that too much has been expected from such courses; they attempt to provide challenging material for the average or near-average students and, at the same time, material easy enough for students in need of extensive remedial instruction. The fact that authors of general mathematics texts have failed for the past 30 years to achieve this dual aim suggests that the goal is an impossible one. What has not been tried, at least to any great extent, are two different courses: one for students in the lowest quartile and another for those in the 25th to

50th percentile range. Because it appeared to present the easier task, the Committee directed its initial efforts toward the latter category.

The committee agreed beforehand that a course for these students should provide opportunities for genuine mathematical discovery at the maturity level of the students and should stimulate them to continue their study of mathematics in the regular sequence. To do this, the content must be carefully chosen. It should not depend on a prolonged systematic development of a mathematical idea; rather, the units should be quite discrete in content and as independent of each other as possible. The content should appear to the student as being obviously relevant to the physical world and worth learning about, so the applied aspects of mathematics should be stressed. Presenting new material constantly rather than halting entirely for a review of old topics also helps. In addition, the text should suggest activities involving the use of the students' minds and hands in a creative fashion, and should provide for drill as an integral part of the successful completion of the activity. The specific learning objectives should be to extend student understanding of principles in the areas of number, operation, measurement and approximation, function and relation, proof, symbolic representation, probability and selections, and problem-solving.

The Committee suggested units on the following topics, but left the final selection to the writing team:

Geometry
Mathematical Thinking in Geometry
Mathematics of Selection and Arrangements
Mathematics of Chance
Statistics
Ratio and Proportion and Their Uses
Measurement: Direct and Indirect
Numeration Systems
Arithmetic from an Advanced Point of View
Nomographs
Patterns, Formulas, Graphing Data
Equations and Inequations

When the writers first assembled, on the University of Oregon Campus in Eugene in the summer of 1963, the sequence for each chapter was finally determined. Each section of each chapter was to include a preliminary paragraph to orient students for the class discussion exercises, which were to follow that paragraph immediately and which were to comprise the most important part of the text. Each section was also to contain exercises for students to work on individually during a supervised study period or at home. Since students were to "discover" the mathematics they were to learn, the exposition in the student text was to be kept to a minimum; and, instead, the bulk of the exposition was to be included in the accompanying teacher's commentary.

The writers stayed together for 5 weeks, and during this time wrote nine chapters. A tenth chapter, on statistics, is to be included when a revision is made. The teacher's commentaries were also prepared, but their content was mainly answers to exercises in the student text; lack of time had prevented inclusion of a great deal of exposition.

EXPERIENCES IN MATHEMATICAL DISCOVERY: PRELIMINARY EVALUATION

by

Emil J. Berger*

In December 1962, the Board of Directors of the National Council of Teachers of Mathematics (NCTM) approved an expenditure of $40,600 to finance a General Mathematics Writing Project to produce text materials for 9th-grade students in the 25th to 50th percentile range in mathematics achievement. This action was in accordance with a recommendation from the NCTM Committee on Mathematics for the Noncollege Bound (MNCB), which had worked out a comprehensive plan for production of text materials for six different categories of noncollege bound students, and had recommended giving priority to preparation of materials for this particular category of 9th-grade students. The General Mathematics Writing Project was put under the direction of Dr. Oscar Schaaf of Eugene, Oreg., and an Advisory Committee for the project was appointed by the president of NCTM.

From July 1 through August 2, 1963, a team of 12 writers that included 2 mathematicians produced the preliminary edition of the text, which is entitled *Experiences in Mathematical Discovery (EMD)*. The preliminary edition is multilithed and is bound in two volumes. A *Teacher's Commentary*, duplicated by the ditto process, accompanies the text.

At present the preliminary edition of *EMD* is being evaluated. In particular, answers are being sought to four major questions:

> How effective is *EMD* as compared with conventional 9th-grade general mathematics materials?
>
> What do teachers think of *EMD* as instructional text materials for students of 9th-grade general mathematics?

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> What do mathematicians and educators think of *EMD* as a text for 9th-grade students of general mathematics?
>
> How readable is *EMD?* (Since *EMD* was written to be read *by* students and not merely to them, it seems desirable to make some determination of its readability.)

Since *EMD* was written expressly for 9th-grade students with mathematics achievement in the 25th to 50th percentile range, the evaluation is being carried out with students normally registered for 9th-grade general mathematics. This is believed to be the best available approximation of the population for which the text is intended.

The first of the four major questions calls for a comparison between students using *EMD* and students using conventional 9th-grade general mathematics textbooks. A brief search was conducted to identify teachers who were scheduled to teach two typical classes of 9th-grade general mathematics during the school year 1963–64 who were willing to use *EMD* in one of these classes and a conventional textbook in the other class. Forty-five teachers responded, and 35 of them completed the initial phase of the required testing program, which will be described subsequently. At present, this aspect of the evaluation is centered around 994 students who are trying out *EMD* and 985 students who are using conventional 9th-grade general mathematics textbooks. Answers are being sought to the following specific subquestions:

> Is there a significant difference in achievement between the two groups of students?
>
> Are there significant differences in achievement among students using different conventional textbooks?
>
> Is there a significant difference in attitude change between the two groups of students?
>
> Is there a significant relationship between change of attitude and student achievement?

Obviously, whether or not answers can be obtained to the foregoing questions with the sample that is being used will depend on the comparability of the two groups of students, which cannot be determined until after the data-gathering has been completed. The reason for proceeding in this way is that it was not possible to assign subjects randomly to the two treatments.

To answer the specific subquestions posed above, the following testing program is being carried out with both groups of students:

> To obtain a measure of student attitude change for the year, Hoyt's *Mathematics Inventory Test* was administered last fall as a pre-test and will be re-administered this spring as a post-test. (This test is designed to provide a measure of student attitude toward mathematics.)

> To obtain a measure of the initial ability of each student, the *School and College Ability Test* (*SCAT*), Form 3A, was administered last fall.
>
> To obtain a measure of the initial level of mathematical skills and understandings of each students, the *Sequential Test of Educational Progress* (*STEP*), Form 3A, for mathematics was administered last fall.
>
> To obtain a measure of student achievement for the year, two different tests will be administered in the spring. One of these is the *Sequential Test of Educational Progress* (*STEP*), Form 38, for mathematics, and the other is an achievement test based on the content of *EMD* but couched in "neutral" language. The Advisory Committee for the General Mathematics Writing Project is responsible for the construction of the latter test.

To determine how effective *EMD* is as compared with conventional 9th-grade general mathematics materials, the following analyses of the test data will be made:

> An analysis of covariance of achievement as measured by the difference of pre-test and post-test *STEP* scores with adjustment made for variations in student ability as measured by *SCAT* scores. (The purpose of this analysis is to determine whether the difference between the means of students using *EMD* and those using conventional textbooks is significant.)
>
> An analysis of covariance of achievement as measured by the achievement test based on the content of *EMD* with adjustment made for initial ability as measured by *SCAT*.
>
> An analysis similar to the one above describing textbooks will be carried out to determine whether there are significant differences among the means of students using different conventional textbooks.
>
> An analysis of variance of attitude-change scores for students using *EMD* and students using convential textbooks.
>
> Computation of a product-moment coefficient of correlation to determine to what extent attitude change correlates with student achievement. (Such a coefficient will be computed both for students using *EMD* and for students using conventional textbooks.)

To obtain information on the second major question—What do teachers think of *EMD?*—each teacher using *EMD* has been asked to complete a "Chapter Report" after using each chapter. Specifically, the teacher is asked to rate the material in each section of each chapter as "Good," "Fair," or "Poor," and to comment briefly on the material. A second part of the "Chapter Report" calls for an evaluation of student reaction and an indication of the teacher's reaction to the chapter as a whole.

"Chapter Reports" received thus far indicate that teachers are favorably disposed to *EMD*. However, a review of the completed "Chapter Reports" that have been returned indicates that the teachers are relatively noncommital about the choice of topics and the mathematics contained in the text. The tryout teachers seem to regard these matters as not open to question. In the main, teachers' comments may be divided into those dealing with student interest and motivation, those dealing with the difficulty of the material (e.g., "too hard" or "too easy"), and those dealing with organizational aspects of the text. Very few of the "Chapter Reports" include any comments about mathematical accuracy, suggested alternate treatments, and so on.

To gain some appreciation of the opinions of mathematicians and educators on *EMD*, about 15 such people were asked to submit chapter-by-chapter appraisals of the text. The nine responses received were from three college mathematicians, four education specialists from colleges of education, one State mathematics supervisor, and one high school mathematics department chairman. Subsequently, three members of the Advisory Committee—one specialist from a college of education, and two high school teachers of general mathematics—reviewed the appraisals and drew up a summary of them that suggests that mathematicians and educators believe that *EMD*, in its present form, places too much emphasis on the discovery approach and that more formalization of those mathematical concepts that students are expected to discover may be needed. The reviewing group also felt that *EMD* may need some strengthening in the development of basic mathematical concepts before it is published in final form.

The last major question in the evaluation of *EMD* is that of readability. To obtain reliable information on reading ease (or difficulty), Robert Jackson of the University of Minnesota was engaged to check the readability of *EMD*, using the Flesch reading-ease formula adapted for mathematics materials. Reading-ease scores were computed for three to six randomly selected samples of 100 words in the explanatory material in each of the nine chapters.

Jackson reports that there is a wide diversity of reading difficulty within chapters. For example, the samples selected from Chapter V extend from a 7.5 grade level to an 11.5 grade level, and those chosen from Chapter VII from a 6.5 grade level to a 14.0 grade level. Mean reading scores for all chapters range from an 8.0 grade level for Chapters I and II to an 11.5 grade level for Chapter VI. The mean reading-ease score for the total text (for explanatory materials) is about at the 9.0 grade level. In conclusion, Jackson recommends that for the ability level of the student for whom this test is intended, the reading-ease scores should probably be between the 7.0 and 8.0 grade levels.

COUNTERING CULTURAL DEPRIVATION VIA THE ELEMENTARY MATHEMATICS LABORATORY

by

Lore Rasmussen*

It is difficult for college-educated, middle-class teachers to imagine the deprivation which the slum child faces from birth. We are most aware of his meager comprehension, limited speaking vocabulary, and lack of fluency in speech. As mathematics teachers, we must pay equal attention to his paucity of experience with manipulation of the objects of the physical world.

His family lives in crowded quarters, and owns few artifacts of our culture. The child has few toys and possessions to play with, to hoard, to collect, to classify. Neither does he have strong geographic roots in a neighborhood which could have given him a secure space orientation, since his family is continuously on the move.

If he lives in the center of a large city, he has missed learning the concept of "many" as formed by observation of the many leaves or needles of a tree, pebbles near a brook, or grass blades in a meadow. Although he has, instead, impressions of many houses, windows, cars, streets, and people, he cannot own, hoard, rearrange, and sort these as he could do with leaves or pebbles. And how can anyone—without playful, pensive, active involvement of the senses in the physical environment—gather mastery over its properties? The environment must of necessity overwhelm us if we cannot understand what it can do for us and how we can manipulate it and create with it.

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This is why teachers should experiment with laboratory environments for teaching mathematics to the culturally deprived low achievers. Through the laboratory we can create in the schools selective, planned, compensatory environments in which the child may have the primary, active sensory experiences from which generalizations and abstractions can be made and concepts formed. Only after we have laid the experience base by affording to the child exploration through action can we fruitfully begin to combine action with representation, or symbolism. Only after this codification of experience is mastered can the real search for structure begin.

What Is an Elementary Mathematics Laboratory?

To the child it is a playroom where things can be counted, moved, rearranged, stacked, packed, wrapped, measured, joined, partitioned. It is a room with things to be weighed and weighing instruments, machines with buttons, levers, and cranks that count, record, and project; with books for browsing, books for reading, and empty books to be written into; with objects of interesting shapes and many sizes to be used for building and comparing.

To the teacher it is a planned environment in which one puts children, so that each question one poses to them about numbers or shapes can be illustrated in innumerable ways. It is a room in which the children can have the best possible support for problem-solving through concrete experiences and readily available tools, and where they can verify through experiments the correctness of their answers independently of the teacher.

Who Is the Lab Teacher?

Regardless of her amount of formal training in mathematics, she remains an active and successful learner of this subject and a teacher-specialist. She learns mathematics not only from courses, but through independent study and through the problems children formulate and solve in original ways, and she searches for mathematical structure in other subject matter areas as well. She believes in the learning capacity of her charges, and if she were to voice aloud a personal creed, she might express it as follows:

"I believe that the urge toward mathematical thought and analysis is a universal characteristic of all healthy human beings.

Environmental conditions can suppress this urge, but cannot kill it permanently except under the most adverse circumstances. I must find ways to satisfy the natural curiosity of all children about numbers and shapes and to create the educational conditions which stimulate thought.

"I believe that mathematics is not only necessary for survival in a technological society, not only the language of commerce and science, but also that it is beautiful and that it is an intensely satisfying private activity. Its logic and internal consistency affirm the sanity of the universe and of man.

"I believe that the pupil *wants* to learn and *can* learn, but to help him do so, I must learn to communicate with him in the language and experience he understands.

"In an abundant society built on humanitarian values, children are the greatest resource. Money is expendable, materials are created to be used, and equipment is meant to become obsolete. The children, however, are to be nurtured and educated; the children are not expendable."

Things for the Mathematics Laboratory

Myriads of things are needed in the mathematics laboratory: raw materials from which things can be made by children—sand, water, clay, pebbles, and cleverly designed graduated containers to hold them.

String, paper, tape, dowels, pipe cleaners, and hollow tubes; solids, such as spheres, rectangular prisms, tetrahedrons, cylinders, cones, etc.; tin cans, food containers, paper cups, balls; and precision-made, carefully related geometric models, sets of blocks of wood, plastic, etc., hollow blocks with lids and solid blocks to fit into the hollow blocks, and blocks with holes drilled at various angles and dowels to fit; tiling surfaces and tiles in many colors and shapes, and adhesives to hold things together temporarily; pencils, felt pens, paint, chalk, and paper (ruled, squared, plain), blackboards, slates, tabletops, floor surfaces to be written on; scissors, paper cutters, rulers, drafting tools, pegboards and pegs, nail boards, and rubber bands with many kinds of arrays allowing quick experiences in grouping and in making polygons.

Mathematician's tools and labor-saving devices: clocks and timers, rulers, yardsticks, metersticks, thermometers (Centrigrade, Fahrenheit, and uncalibrated), scales and balances, slide rules, adding machines (nonelectric), pocket and desk calculators, opaque projectors, compasses and protractors, blueprints, play money, magnifying glasses, dice games (strategy and chance), puzzles, pictures, books, numberlines, Cartesian coordinate systems (on blackboard or floor).

Books and other printed matter: a library of attractive trade books on mathematics for children; picture books on design, architecture, mechanical drawing; science experiments with strong mathematical implications; and maps, charts, globes, mail order catalogs, cookbooks, and secret code books; textbooks of various difficulty levels; loose worksheets, unit booklets for independent written work filed according to topic and level of difficulty (bought commercially and/or developed by the teacher and groups of children over a period of years); self-checking, self-correcting guides, hint books, answer books, and self-tests; programed instructional material, teaching machines, file boxes of cards with short questions, organized by level of difficulty and by topic.

Spatial Design of the Mathematics Laboratory

The room must have:

Space for large-group, teacher-centered activities; it should include a large blackboard area.

Space for small-group activities; it may be either table-centered or floor-centered.

Space for utter privacy, protected from noise and interference; a library section with carrel-type tables is ideal.

Space for exhibition of completed work—shelf space, wall space, or floor space.

Space for storage of work in progress—shelf space, closet space, or drawer space.

Open space for walking games, rhythm games, "geometric dance patterns."

How the Mathematics Laboratory Should Be Conducted

The mathematics laboratory is sometimes a silent place because at times the teacher expects the children to work without any communication with one another. Most of the time, it is filled with "working noises" which purposeful adults make, too, as they confer, consult, assist, praise, and criticize the work of their coworkers.

The teacher is in control, but tries to involve the children in setting standards of behavior and in planning the work. She alternates quiet individual work periods with free activity periods, lively game sessions, mental arithmetic sessions, etc. She lectures little and keeps whole-group teaching to a minimum, acting instead as a small-group or individual tutor. She stimulates all through appropriate questions, assists the discouraged, directs the learning sequence, and provides evaluation.

PART III

RECOMMENDATIONS

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RECOMMENDATIONS

Never before has this Nation been more aware of the conditions which contribute to the inability of a significant number of our population to seek and maintain gainful employment. The consensus of this conference was that the solution of this persistent problem rests largely in an improved educational program in its broadest concept. After the conference, the Office of Education staff wrote up the following summary of specific recommendations that emerged from the conference discussions. These were not all the recommendations, although the ones selected here reflect the main ideas proposed in the discussion sessions. Few of the recommendations had unanimous support from the conference participants, but it is hoped that this presentation of them will stimulate improvement of teaching for the low achiever in mathematics.

A. Establish a National Commission on Mathematics Education for Low Achievers

This commission should be the result of cooperation among representatives from industry, labor, professional organizations, mathematicians, teachers, school administrators, and government agencies. The commission would establish goals in mathematics education for low achievers and plan ways of reaching these goals.

Some activities of the proposed national commission might be:

(1) *To establish a clearinghouse of promising practices in teaching mathematics to low achievers.*

The clearinghouse might be maintained by a professional organization such as the Association of State Supervisors of Mathematics, the National Council of Teachers of Mathematics, or a Federal agency such as the Office of Education.

(2) *To publish a yearbook on problems of teaching mathematics to low achievers.*

The role of the commission might be to organize writing teams or to contract with individuals.

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(3) *To promote cooperation between agencies.*

Because educational problems cannot be solved effectively without measures to improve the living conditions of the students, cooperation is necessary between educators and agencies responsible for neighborhood improvement. Mathematics educators can make special contributions by giving advice on the learning environment, such as properly equipped mathematics laboratories in secondary schools, in elementary schools, and in experimental nursery schools; and by developing teaching programs adapted to the special needs of the low achievers, taking full advantage of the unique ways in which mathematics can help these youngsters, both immediately and in the long run.

(4) *To set up a national conference to study the problems of marking, evaluation, and promotion.*

There are many unresolved questions about the evaluation of pupil achievement, especially at the low end of the scale.

(5) *To establish summer camps for low achievers.*

Take students from slum and low economic backgrounds and teach them in the organized environment of a summer camp. Study the results and report findings.

(6) *To establish lines of communication with industry, business, governmental agencies, and educational agencies.*

The communication effort would make available the resources of the mathematical community wherever assistance is needed to develop appropriate educational programs and materials. At the same time, much information would flow from business, industry, and government so that educators are in close touch with the world of work.

B. Establish Research and Development Centers

What is needed is substantial support for a long-range research and development program for the study of low achievers in mathematics on a variety of fronts. Since this work requires a combination of talents not available in any one place, mechanisms must be devised for experts to communicate their ideas to each other and for adequate planning before a large-scale research and development program could be effective. Any such program must have some provision for field work under practical conditions. The needs include the following:

1. Basic psychological and educational research.
2. Curriculum experimentation.
3. Design of special equipment, materials, and environments.
4. Consideration of the characteristics and special training of the teachers.

A first step in such a substantial research and development effort might be to establish a summer institute program to bring together experts in the relevant fields. Participants would include mathematicians, teachers, vocational educators, teacher educators, psychologists, sociologists, social workers, industrial training directors, and children.

In the research and development centers, the mathematicians and master teachers would write material and try it out with the low achievers in the center. Psychologists in the centers could study effective learning procedures. The results of the experimentation and controlled instruction could be publicized through publications and through demonstrations at the center. Some centers might include dormitories for students.

Effective methods of teaching selected groups of low achievers would be studied in the research and demonstration centers with the aid of competent psychologists. Identification should be made of the types of low achievers. The mathematical concepts and skills that are best suited for instruction by certain methods and with certain aids should be determined for each type of low achiever. The methods employed with individuals should vary from those employed with very large groups. There should be a variety of aids such as television, films, and programed learning materials.

C. Extend the Research Effort: Some Suggested Studies

During the discussions many particular problems were identified as appropriate for specific research studies but not necessarily as a part of the efforts of the research and development centers indicated above. The following brief list reflects some of the ideas set forth by the conferees. The list is not intended to be comprehensive; rather it is hoped that it will be suggestive to individual researchers who wish to carry out small independent studies.

(1) ***Study the effectiveness of mathematics teachers who have been specially prepared to teach low achievers.***

This study might investigate whether special preparation to teach low achievers increases ability to impart learning to students in this category. The students' achievement could be evaluated and compared with that of students whose teachers had no such special preparation.

(2) *Study means of communicating with parents of low achievers.*

It is difficult to get most parents of low achievers to attend parent-teacher meetings. Investigate other ways of communicating with parents of low achievers.

(3) *Study the effectiveness of teacher assistants in classes of low achievers.*

Many school systems have found the work of teacher assistants extremely effective. Such persons help to prepare learning aids, proctor tests, arrange for films, and do all kinds of clerical work.

(4) *Set up an experiment to study the effects on low achievers of departmentalization in mathematics in grades 4, 5, and 6.*

This experiment could be similar to that conducted a few years ago by the American Association for the Advancement of Science in which the achievement (in all areas) of students who had special teachers of mathematics and science was compared with the achievement of students in self-contained classrooms.

(5) *Study the nature of poverty of experience.*

Poverty of experience seems to be one of the causes of low achievement. Daily experiences of low achievers could be analyzed and contrasted with the experiences of average children. Out-of-school influences could be studied and, also, ways in which the school can make up for deficiencies.

(6) *Study the effectiveness of the discovery approach.*

(7) *Study the effectiveness of the use of games.*

D. Develop Inservice Programs for Teachers

A large-scale program is needed to provide inservice education for many teachers who are now teaching low achievers. This program could be organized at the local school level with resource persons brought in from industry, foundations, educational institutions, government, and other appropriate agencies. The seminar approach is suggested.

In support of the large-scale inservice effort, a small-scale program is needed to provide consultant help to those who are now administering educational programs for low achievers. A first goal might be to produce at least one consultant for each city of more than 100,000 population. Some institutes supported by the National Science Foundation or the U.S. Office of Education might concentrate on this particular program. Universities and colleges should develop special courses in the teaching of mathematics to low achievers. A corps of teachers with special talent for teaching mathematics to low achievers should be identified by the professional organizations. These talented teachers could contribute to the inservice effort in a unique way (e.g., demonstration lessons or television presentations).

APPENDIXES

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A. GUIDELINES FOR ADMINISTRATIVE PROVISIONS FOR THE LOW ACHIEVER IN MATHEMATICS*

The school administrator is a most important factor in the success or failure of a program for the low achiever in mathematics; it is he who must initiate and provide for an effective orientation program for the staff and the community. He must see to it that teachers of low achievers are given proper recognition and special acknowledgement for the importance of their work. He bears the responsibility of drawing leaders in the community in business, commerce, and industry into the development of a program that will prepare the low achievers for a productive role in the community. He has the job of interpreting the program to the community. The following guidelines for the administrators emerged from the discussion sessions:

1. *The administrator should provide mathematics courses for each year (K–12) for the low achiever.*

Schools should strive to offer equal educational opportunity for all and to promote normal growth for every individual according to his capacity. This is a challenge to provide appropriate courses for all pupils at all levels, since all pupils must assume personal responsibilities as well as responsibilities of citizenship. The pupils in the lowest 30 percent are staying in school longer than previously, so an administrator cannot think in terms of terminal mathematics courses for grade 9 or 10. There are certain basic mathematical understandings that begin in kindergarten and must be expanded throughout the child's school career. It is necessary for the low achievers to get certain skills from the mathematics courses in high school that will make them good risks for employers. The mathematics in the later courses can be directly related to future work experiences. These courses might range from 2 to 5 periods per week.

2. *The administrator should provide for maximum individual growth of low achievers by careful grouping.*

A thorough analysis should be made of student deficiencies. It is necessary to obtain as much information as possible about the child, especially intellectual factors, socioeconomic conditions, home adjustment, visual and auditory abilities, past mathematical experiences, and emotional adjustment. All these factors must be considered before these pupils are placed in classes for mathematics instruction. This background information must be readily available to the mathematics teacher. The instruction in the class can then be geared toward the characteristics of each group.

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*These guidelines and the two sets that follow are included here because of the many requests for summaries of the three discussion topics.

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3. *The administrator should see that order is maintained in the classrooms of low achievers.*

The administrator is responsible for developing a school environment in which learning can take place. The organized activities of the mathematics classroom may be so foreign or contrary to the past experiences of some of the low achievers that they cannot or will not accept them. In these cases the administrator has the responsibility to place such pupils in a very small class or to provide individual help until they may be placed in a regular class. Special help should be provided for students with extreme behavior problems. These students should not be permitted to remain in the regular classes.

4. *The administrator should provide semiprofessional aides to the teacher of the low achiever.*

The development and use of a variety of special learning materials and audiovisual aids for low achievers constitute a heavy demand for time that can be more economically provided by a semiprofessional assistant. The administrator needs to secure budgetary appropriations to make these resources available to teachers of low-achieving mathematics students.

5. *Citizens, community leaders, and representatives of business, commerce, and industry should be involved in the development of a mathematics program for low achievers.*

It is necessary to make effective use of the special talents, resources, wisdom, and experience of the entire community in evolving a realistic mathematics program for low achievers. The involvement of citizens should provide help with the following:

a. Identification of job opportunities for students of limited ability,
b. Establishment of work-study programs,
c. Identification of pertinent applications of fundamental skills and basic mathematical principles,
d. Improvement in communication between teachers and community resource persons,
e. Orientation to and acceptance of the program by the community.

6. *The school administrator must involve the parents of the low achievers and establish lines of communication between the parents and the school.*

The attitudes and aspirations of the parents influence the achievements of their children. The usual PTA meetings and school conferences fail to reach most parents of low achievers. Some of the possibilities for contacts with them are visits in the home by (a) visiting teachers, (b) regular teachers on released time during vacation periods, (c) social and welfare workers.

7. *Pupils' marks should tell the truth.*

Administrators, parents, and teachers should study the marking, evaluation, and promotion policy, so that all concerned will know the academic achievement of the pupil.

8. *The administrator must select teachers who are competent in mathematics and qualified by both preparation and temperament to teach low achievers.*

Since many low achievers do not understand the meaning of mathematical operations or of concepts, the teacher should be well qualified in mathematics.

Teachers must have special preparation in techniques and procedures for teaching mathematics to low achievers. They must be sensitive to differences in individual capacities and interests of pupils. They must be able to sense their feelings and discover the reasons for their reactions to the learning situation.

Administrators have the responsibility for, and must take the initiative in, informing colleges and universities of the educational requirements for teachers of these pupils. Administrators must strive to have the necessary courses instituted as part of the preservice education of mathematics teachers.

9. *Intensive professional inservice education programs must be provided for teachers of low achievers in mathematics.*

At present, most teachers have not had adequate preservice education to cope with the problems associated with teaching low achievers. Inservice education activities such as workshops, interschool visits, and demonstrations of new and different teaching techniques should be provided. The administrator should seriously consider the following as he plans to optimize mathematics instruction for low achievers:

a. Departmentalizing grades 4-6 and providing specialized teachers. (These specialized teachers may also serve as resource people for teachers of grades K–3.)

b. Providing a mathematics specialist or helping teacher. (This teacher could serve as a resource person for teachers of grades K–6.)

c. **Orienting school staff. (All staff members should be cognizant of the program so that a concerted team effort can be made.)**

10. *Special guidance is needed for special pupils.*

The lowest 30 percent in mathematics achievement are composed of several distinct groups of pupils, such as those working near capacity, the average or above in ability who are underachieving, the emotionally disturbed, and the unmotivated.

To identify these various groups, determine the cause of their problems, and to seek a remedy requires the joint efforts of an able teacher, experts in psychology and social work, and the help of the guidance department. In most schools, counseling, additional guidance, and psychological services will need to be provided. Providing this service is the responsibility of the school administrator.

11. *The administrator must provide opportunities for teachers to engage in research and experimentation.*

The school administrator must encourage research and experimentation with new and different teaching techniques for low achievers in mathematics. Very little is known about methods which are most effective with low achievers. Very few if any follow-up studies have been made of pupils in this category. Answers to these and similar questions might be found by a local mathematics staff.

B. GUIDELINES FOR THE PREPARATION OF INSTRUCTIONAL MATERIALS FOR THE LOW ACHIEVER IN MATHEMATICS

Good instructional materials are sorely needed for below-average students. Since the need for suitable instructional materials for low-achievers in mathematics was evident, the discussion was focused on the question of the production of the needed materials. As a result the following guidelines evolved:

1. *A team of specialists from several disciplines should select the course content.*

National leadership should initiate action to recruit a team of mathematicians, teachers, and psychologists, to develop outlines of the mathematical content of courses for the low achievers.

2. *A new approach should be followed in designing the experimental materials.*

An approach that is imaginative but realistic needs to be taken in determining the curriculum for low achievers in mathematics. A slowed-down version of mathematics for the college-bound student is inappropriate.

3. *Materials should provide for the development of mathematical understandings essential for vocational competence.*

The low achiever in mathematics has a great need to achieve the mathematical skills needed for gainful employment. As jobs become lost to automation, the worker must possess mathematical skills adequate to qualify for admission to the retraining programs for new or different jobs.

4. *Opportunity for success should be a major aim in the design of the learning materials.*

The old adage "nothing succeeds like success" appears particularly relevant. Success plays a most important role in the development of the individual's concept of his own ability. Students need to achieve a measure of success if they are to be stimulated to further effort. Hence materials should be provided which are within the low achiever's level of comprehension and expressed in language meaningful to him.

5. *The learning materials should be graded in content.*

A variety of learning materials should be provided to meet the differing rates of learning. Attention should be directed toward their flexibility and ease of use in achieving the objectives of the program for the low achievers.

6. *Study units for the low achiever should be short.*

The attention span of the majority of these students is shorter than that of most in their age group generally. The student needs the sense of accomplishment derived from completing a task. He profits from the change of pace provided by short units.

7. *Special units for work-study programs should be developed.*

Cooperative work-study programs require learning materials designed for achievement of the skills demanded by families of jobs. Such work-study materials should be sufficiently flexible to enable the student to learn the mathematical operations necessary for satisfactory on-the-job progress.

8. *Materials should provide a varied approach to the development of mathematical concepts.*

The learning materials should provide for many uses of objects, models, audiovisual aids, and manipulation devices, as well as the use of more complicated instruments and learning aids.

9. *Experimental instructional materials should be developed by expert writers.*

Teams of mathematicians and mathematics teachers noted for their expertise in preparing learning materials in mathematics should be selected for this task. Specialists from related disciplines should serve as consultants. Developmental centers should be established for student tryouts and preliminary testing of the materials.

10. *Evaluation of the experimental materials should be conducted in a variety of schools.*

The experimental materials should be tried out with different types of low achievers. Appropriate measuring instruments should be designed and constructed to determine the degree to which the new materials accomplish their purposes.

C. GUIDELINES FOR TEACHING MATHEMATICS TO THE LOW ACHIEVER

Anyone who teaches mathematics should (a) know mathematics, (b) like mathematics, (c) continue to learn mathematics, (d) be able to communicate well with the learner, and (e) understand the learning process. In addition, teachers of low achievers need special knowledge of the psychological and sociological backgrounds of the children. The following guidelines that evolved from the discussions reflect many of the ideas that were suggested for improving the quality of instruction in mathematics for low achievers and potential low achievers:

1. *Low achievers should be taught by able and well-trained teachers.*

A long-range solution to the problem of the teacher shortage includes a bold reorientation of preservice programs for teachers of mathematics, especially at the elementary school level. In addition to adequate preparation in mathematics, prospective teachers should have intensive psycho-pedagogical training focused sharply on the process of learning mathematical ideas. Low achievers, if properly taught, can learn much more mathematics than they ordinarily do learn.

2. *Modern educational technology should be exploited.*

Multisensory aids and new media often add a dimension to provide insight for the low achiever that he does not get from chalkboard or the printed page. For example, the overhead projector furnishes avenues to capture the attention and curiosity of the learner. Student-made transparencies are an asset, for they challenge the learner to organize his ideas; furthermore, this technique gives recognition to the low achiever for his accomplishment. Well-conceived programed instructional material may be effective for low achievers in mathematics.

3. *Classroom activities should be both purposeful and varied.*

Short intervals of group study, discussion, and independent study are generally better for the low achiever than long intervals because the interest span is short. Prolonged periods of unguided activity may cause the learner to practice his errors. The teaching technique to be used depends on the concept to be learned, the individuals involved, and the particular situation at the moment.

Different ways of looking at the same mathematical concept may reinforce the idea or provide the insight needed. Every effort should be made to capitalize on the interest and motivation of the learner through the use of games, puzzles, short cuts, and discovery exercises that arouse curiosity and imagination.

4. *Particularly for the low achiever, the need for mathematics comes from experiences in the physical world.*

Observations of physical events, recording of these events, and the resulting order that mathematics supplies through the organization of data can help the child make sense out of related events in the world about him. The low achiever does this less rapidly and with less sophistication than the high achiever, but the process is the same. Abstraction of the concept from the application may come slowly to the low achiever but it enables him to organize ideas and predict what will happen in new situations. It is folly to strive for abstraction without the base of concrete experience. Vocational goals and out-of-school activities often provide the spark of motivation for learning mathematics.

5. *The teacher should be receptive to questions.*

It is necessary for the teacher to adjust to the questions of the learner. No question or answer is trivial in the learning process. The questions asked by the learner are often the clue to understanding his difficulty. In fact, the formulation of questions is an essential part of the learning process.

6. *A laboratory setting is especially effective for low achievers.*

Evidence from research clearly indicates that active experimentation in which the child handles concrete objects and observes what happens precedes the formal operation stage in learning mathematical ideas. For slum children who come to school with a paucity of experience with manipulation of objects, the elementary school teacher must provide the first selective planned environment in which active sensory experiences can take place. Only after the codification of experience can the real search for structure begin. A laboratory for learning mathematics at the elementary school level should contain raw materials such as sand, water, clay, which can be formed, measured, and distributed; string, paper, dowels, hollow tubes, tape, and pipe cleaners to bend, cut, fold, measure and to be measured by; solids such as prisms, cylinders and containers of all sorts; scissors rulers, pencils, rubber bands; and printed material of various kinds. The teacher plays the central role in creating a learning climate; learning progresses when there is an atmopshere of working with objects, asking questions, playing mathematical games, and discovering new ideas. The discovery-laboratory approach is more a method than a well-defined physical facility.

7. *Special help should be provided for beginning teachers.*

It is a common practice, unfortunately, to assign beginning teachers and new teachers in a school system to the unwanted classes, meaning usually the low achievers. Beginning teachers need special help, particularly someone to turn to in this critical period of their development as teachers. Often their inexperience is coupled with inadequate preparation and perhaps lack of certification in mathematics. Their difficulties are compounded by classes of low achievers. The following specific suggestions are made for beginning teachers:

a. Learn as much mathematics as you can. You must know mathematics unusually well to be able to split up the usual processes into simple elements.

b. Study the recent developments in the curriculum and acquire experience in teaching the new courses. You should know the best mathematics in the best expositions available before you can adapt these ideas to the special needs of low achievers.

c. Request from your administration an adequate supply of manipulative materials, and learn how to use them in teaching low achievers.

d. Become acquainted with what these students are doing in their industrial arts, business education, and home economics classes, and use these courses as sources of applications of mathematics.

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